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<http://db.tt/9PdUuOrq>





# ICoSCIS



## Introduction to Networks



European Territorial Cooperation Programme  
**Greece-Bulgaria 2007-2013**  
INVESTING IN OUR FUTURE

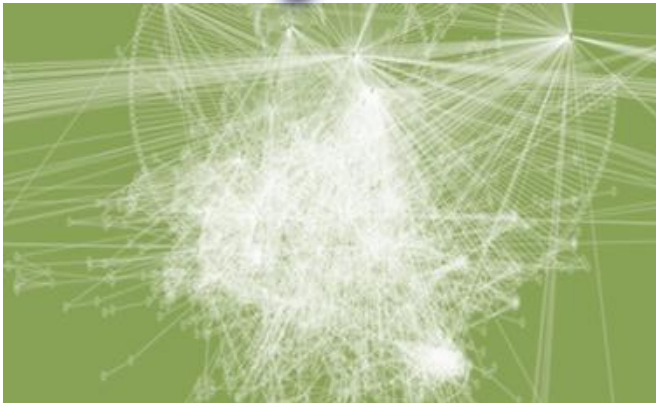
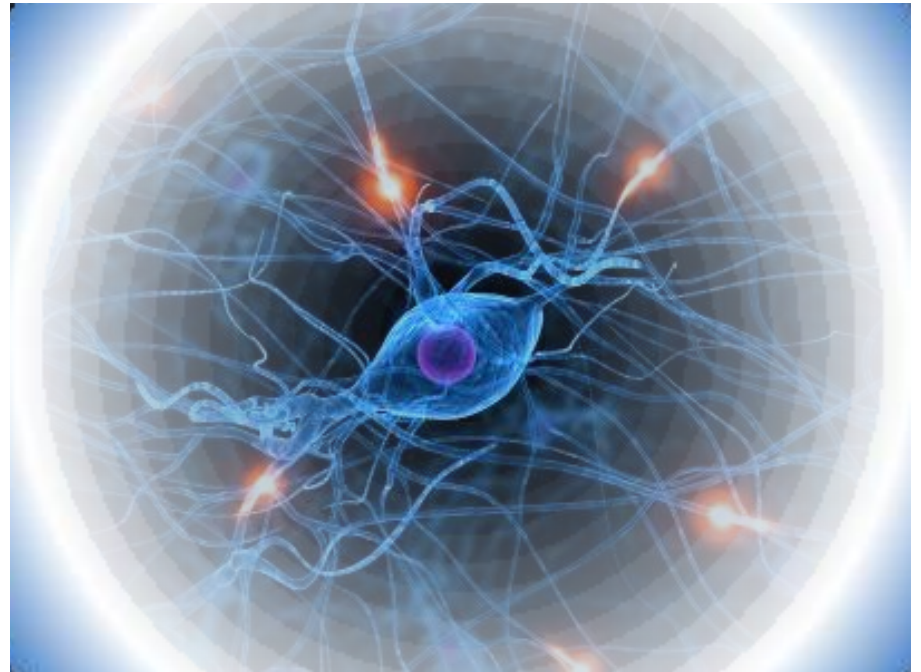
European Territorial Cooperation Programme  
**Greece - Bulgaria 2007 - 2013**

This Project is co-funded by the European Union (ERDF)  
and National Funds of Greece and Bulgaria

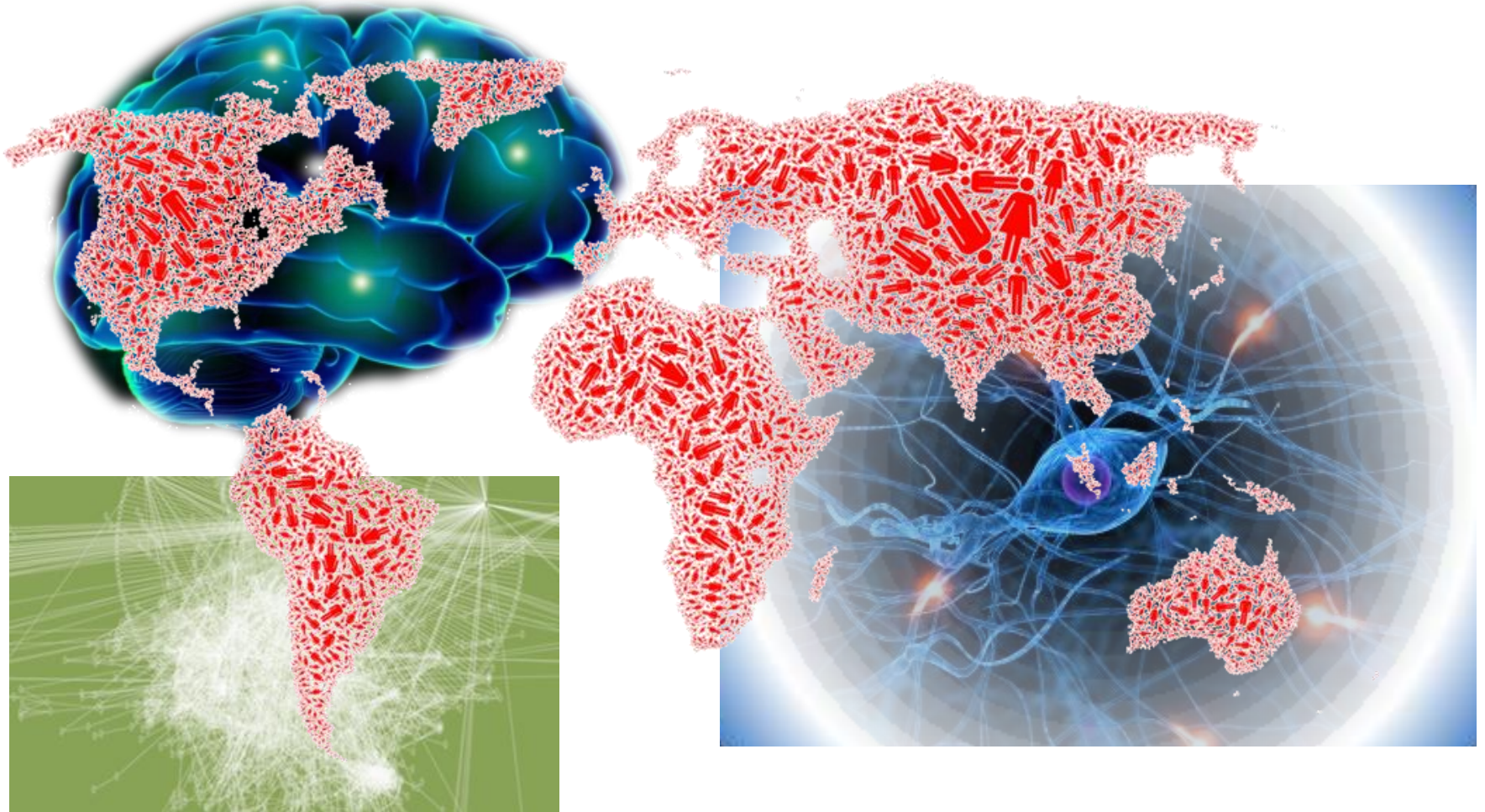


*Blagoevgrad 2013*

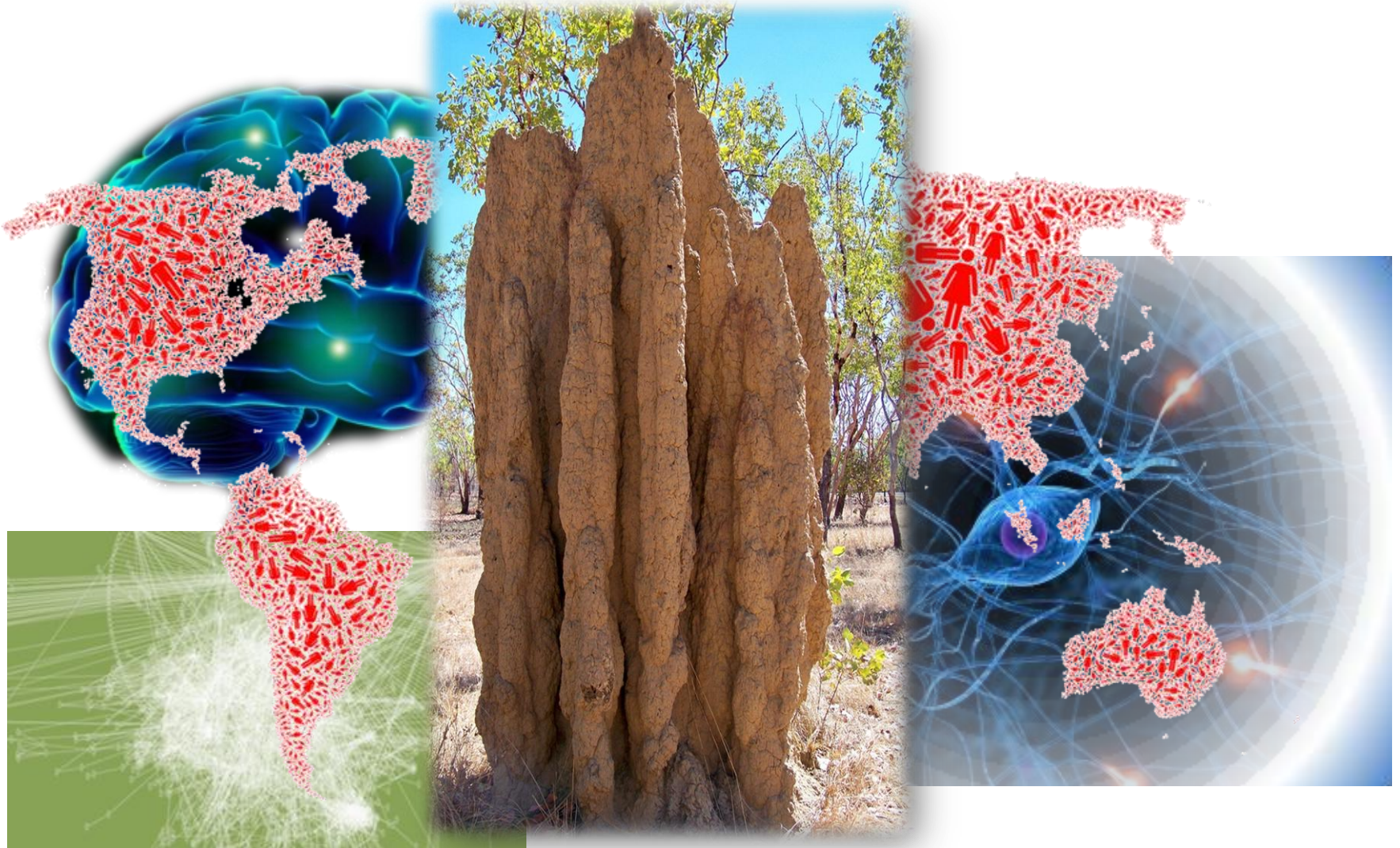
# A small review to complexity



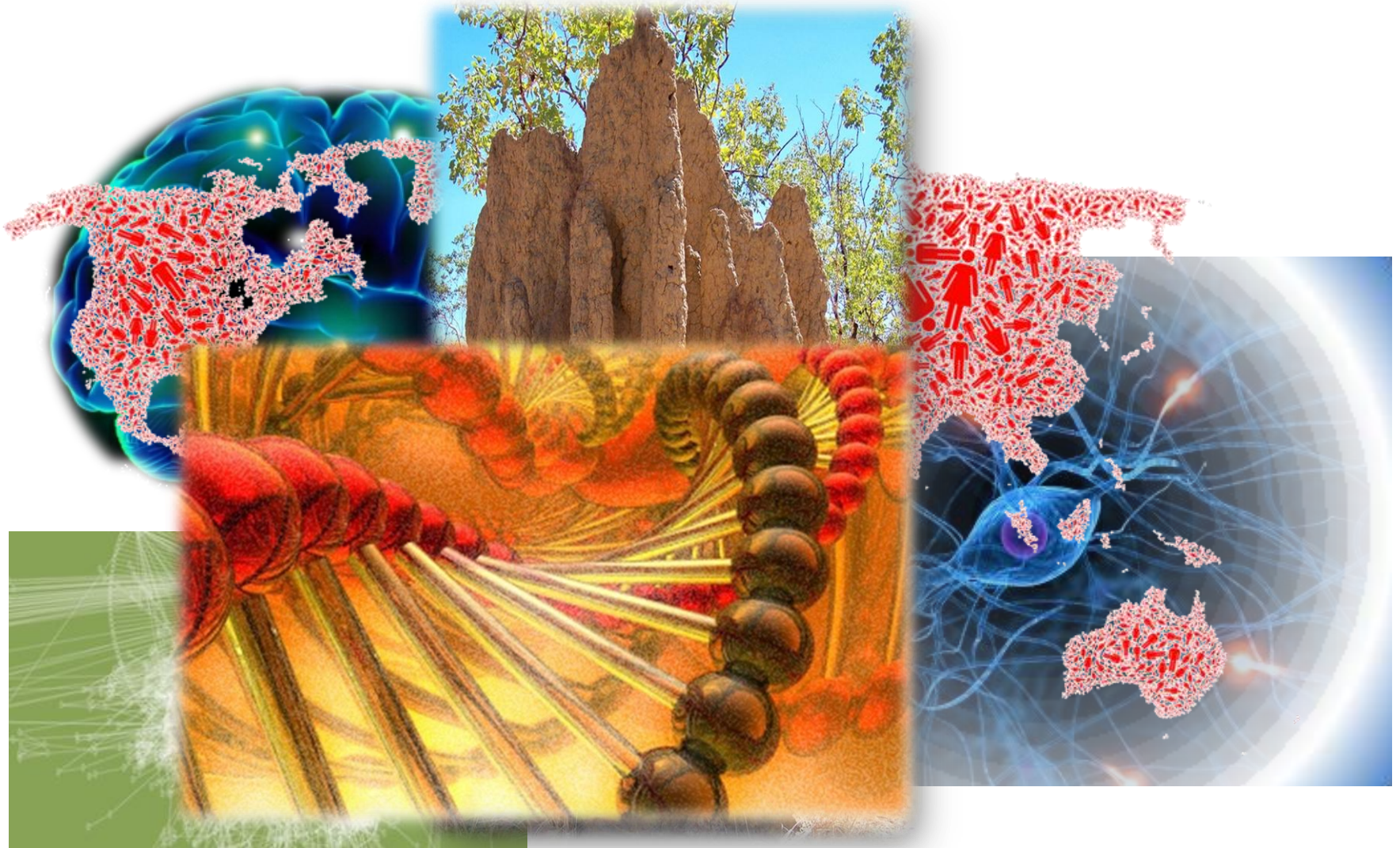
# A small review to complexity



# A small review to complexity



# A small review to complexity



# Modeling with Random Numbers

CPU



Random number Generator



Random number



Independent variable of the problem.

*The truth is that we can NOT have absolutely random numbers!!*

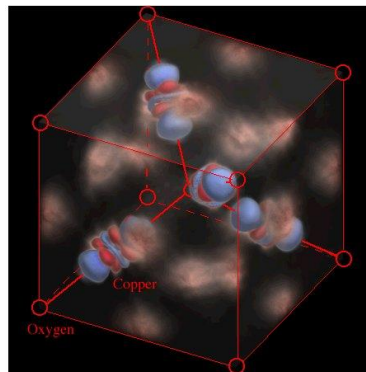
# ...But WHY modeling?

A simulation is a procedure that takes place virtually in a machine.



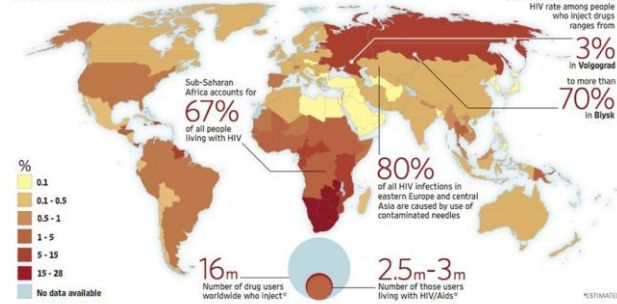
Even because the *timeframe / space* of the experiment is too small or large!  
NOT in human observable size!

Even because the experiment has *not* a human observable parameters.



## THE WORLDWIDE SCOURGE OF HIV/AIDS

Prevalence of HIV/AIDS in adult population



GRAPHIC: CAT DRIVSON; PETER GUEST

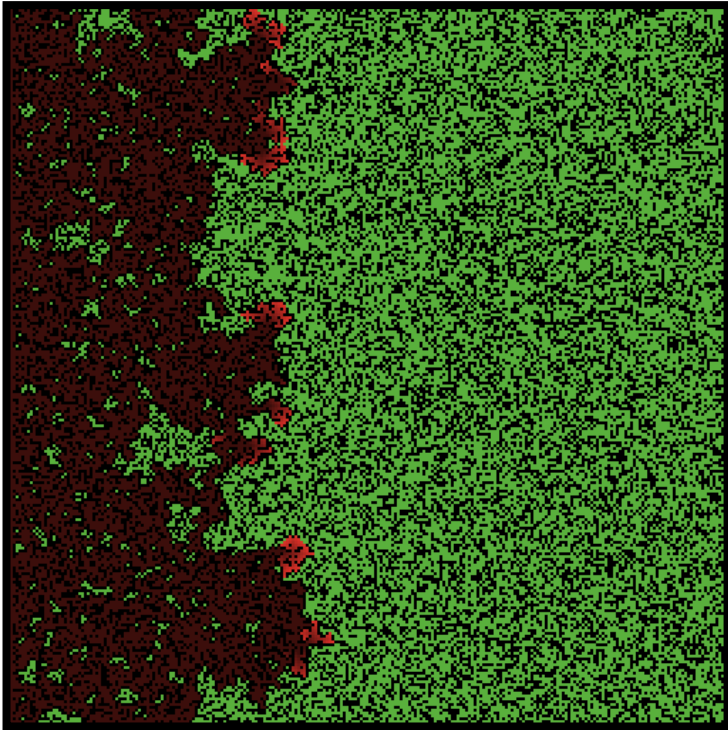
SOURCE: WWW.UNAIDS.ORG





# Percolation

*in real life*

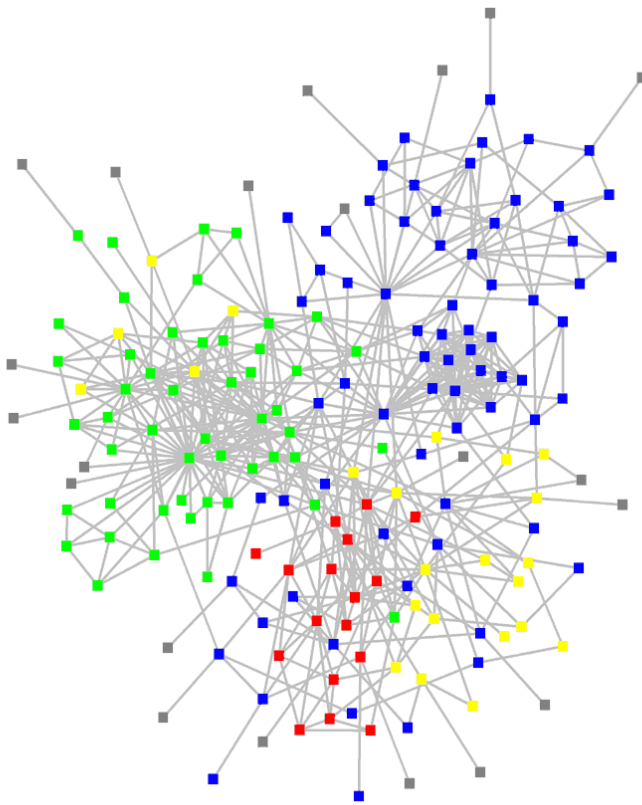


*Forest fire*

*Oil in a porous material*

# Percolation

*in real life*

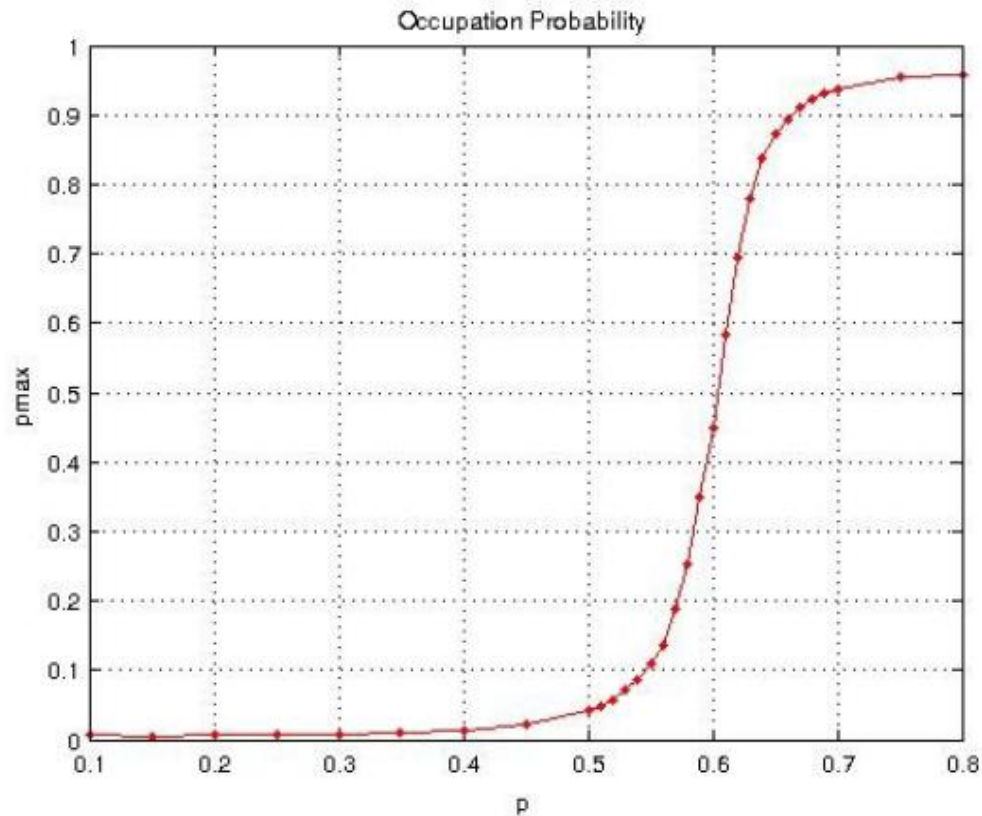


***Spread of a disease  
or  
information***

# Percolation

*in the computer!*

Plot of the probability of one site to belong to the spanning cluster



# A small review to complexity

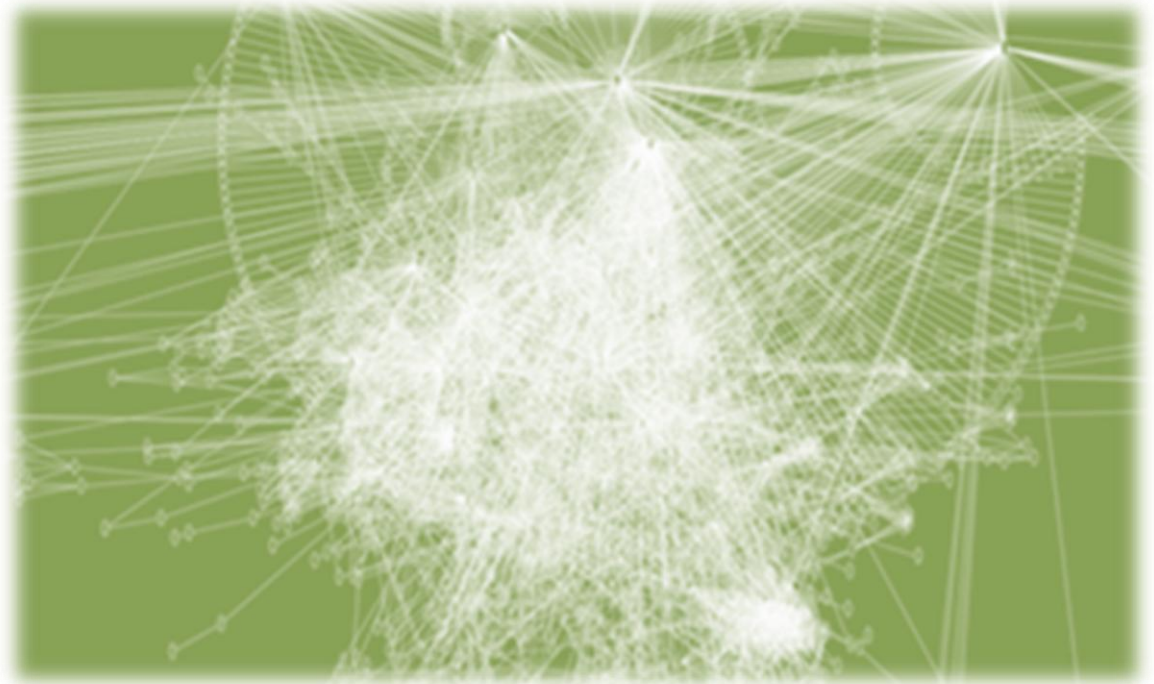
No blueprint or master-mind

Self-organization

Evolution

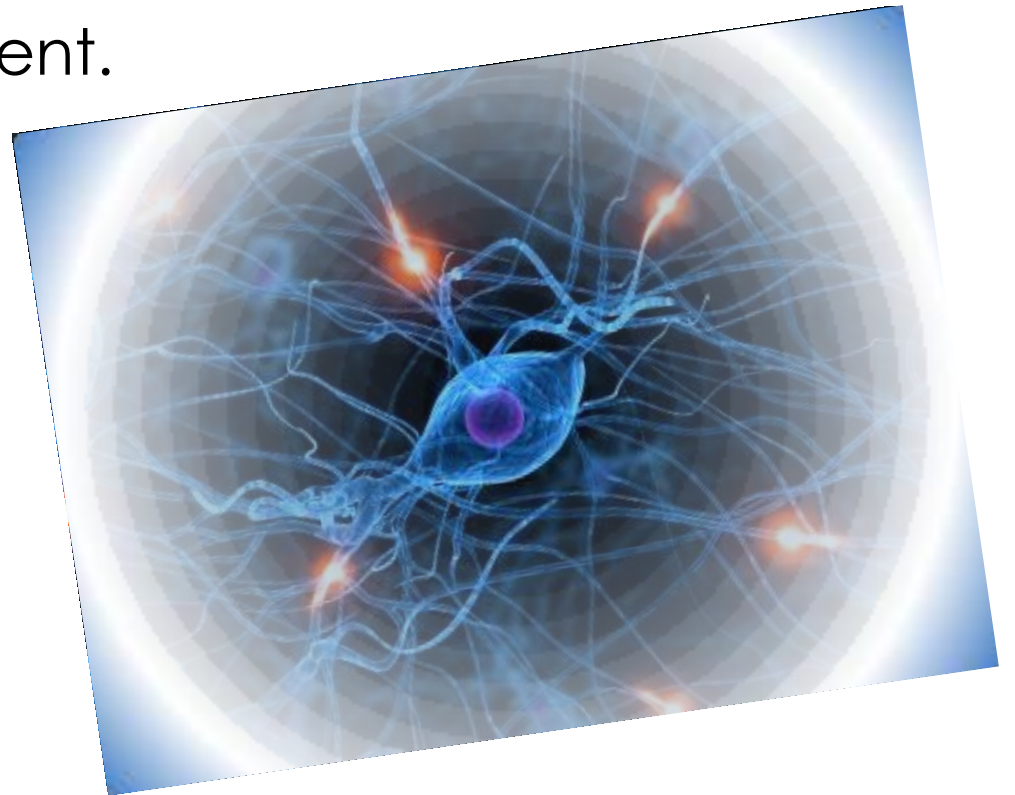
Adaptation

Emergence

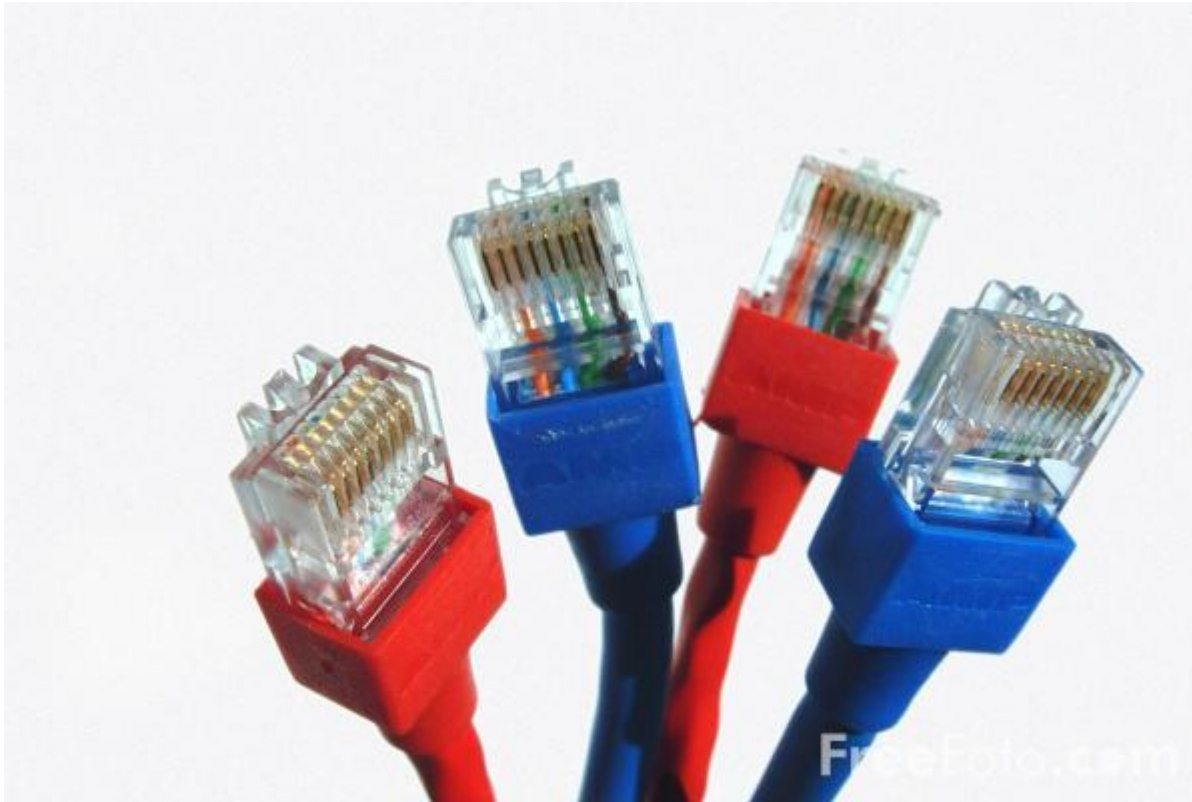


# A small review to complexity

Behind each complex system there is a network, that defines the interactions between the component.

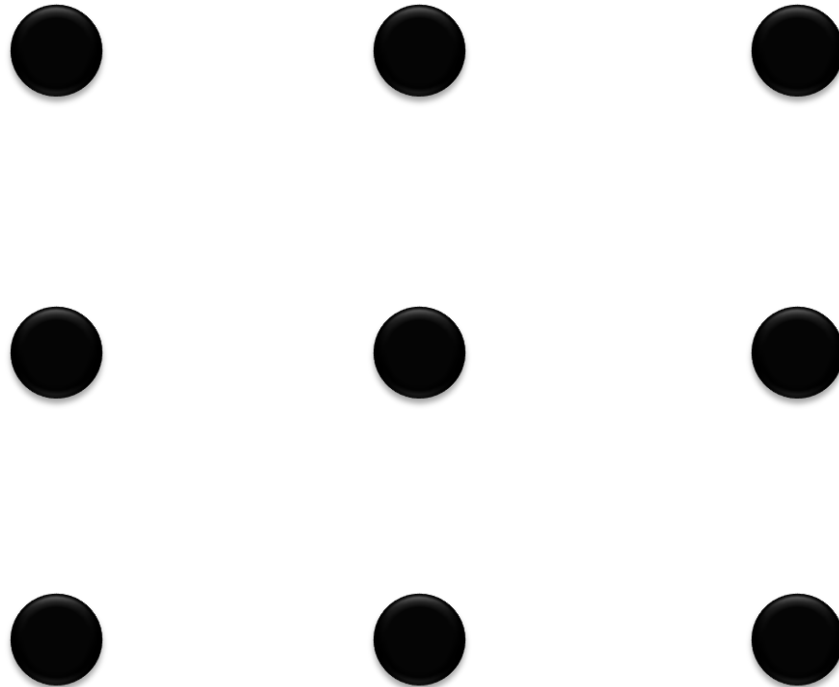


# What is a network ?



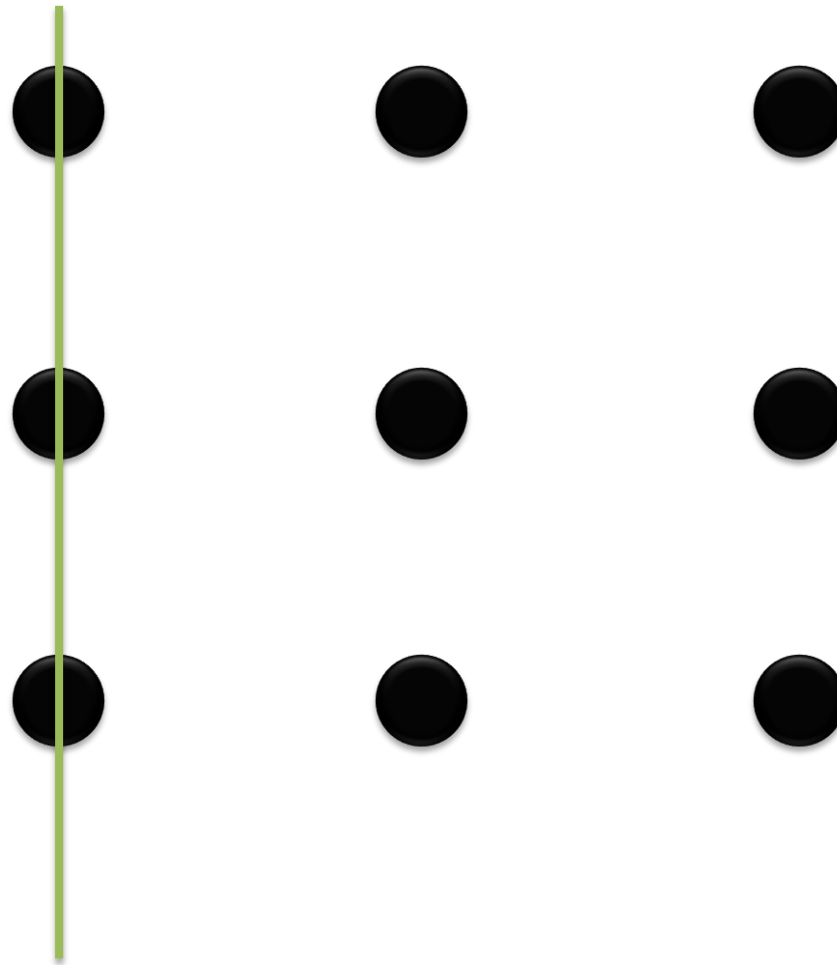
# Let's play a game!

**Connect all the dots with 4 continuous straight lines.**



# Let's play a game!

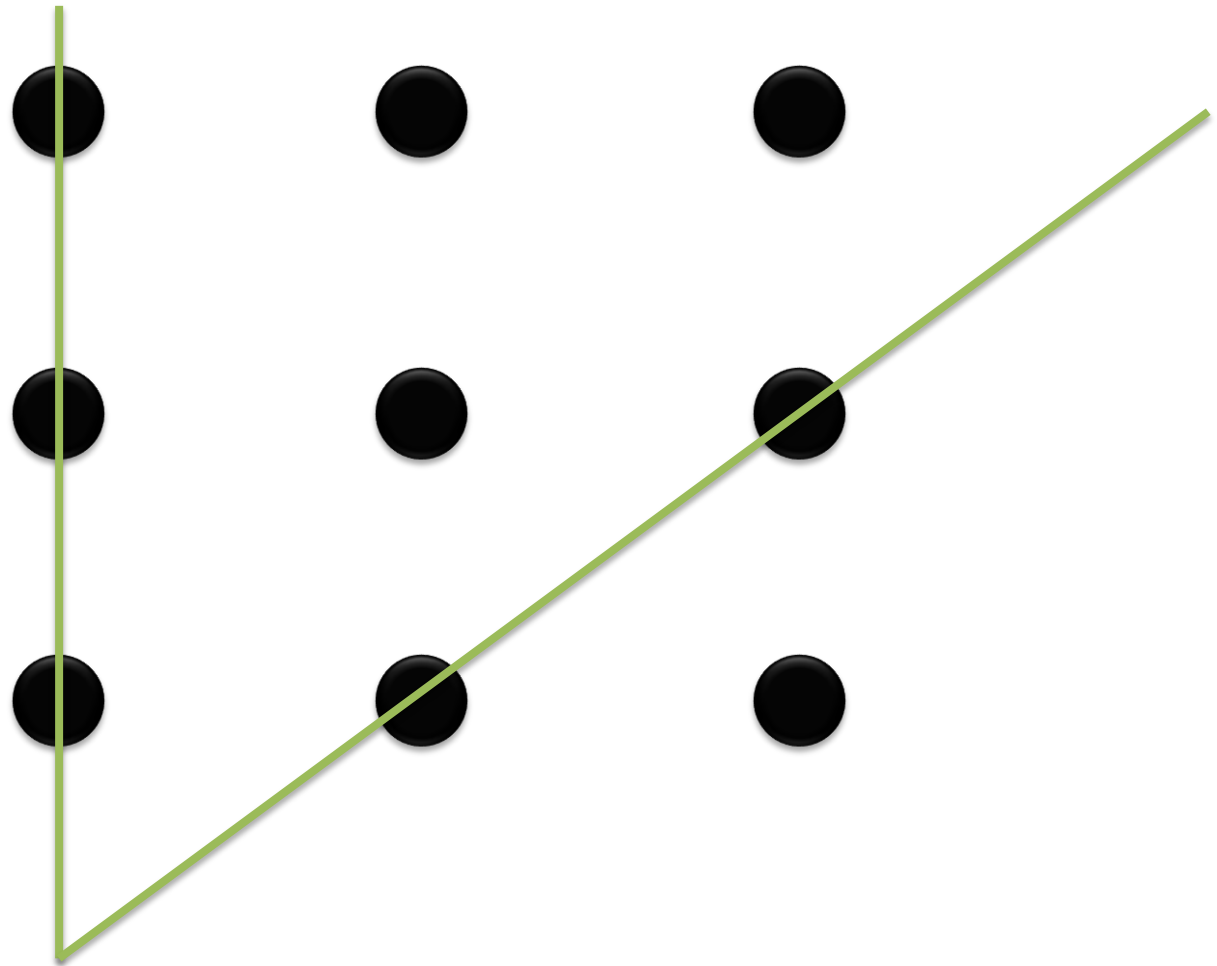
Connect all the dots with 4 continuous straight lines.





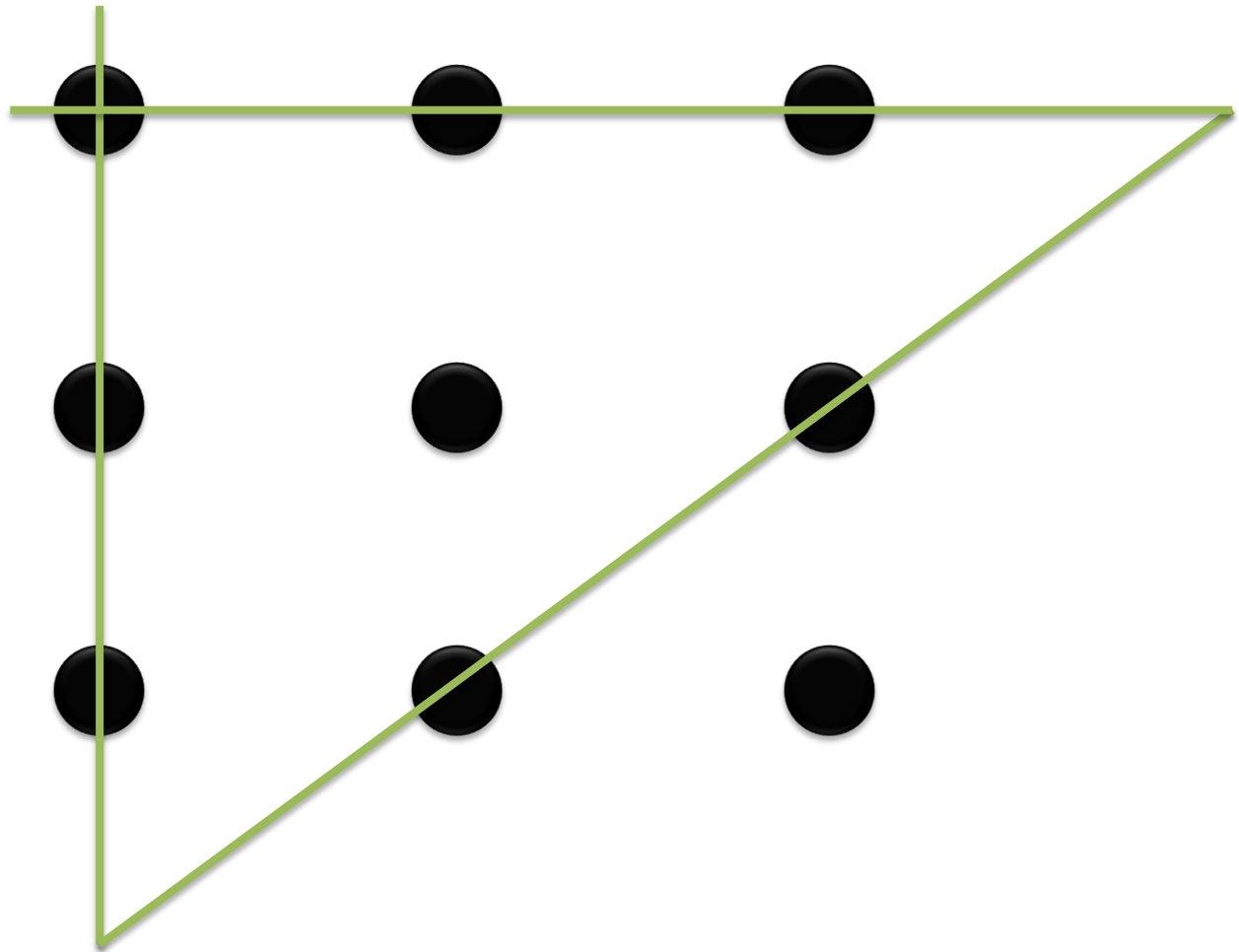
# Let's play a game!

Connect all the dots with 4 continuous straight lines.



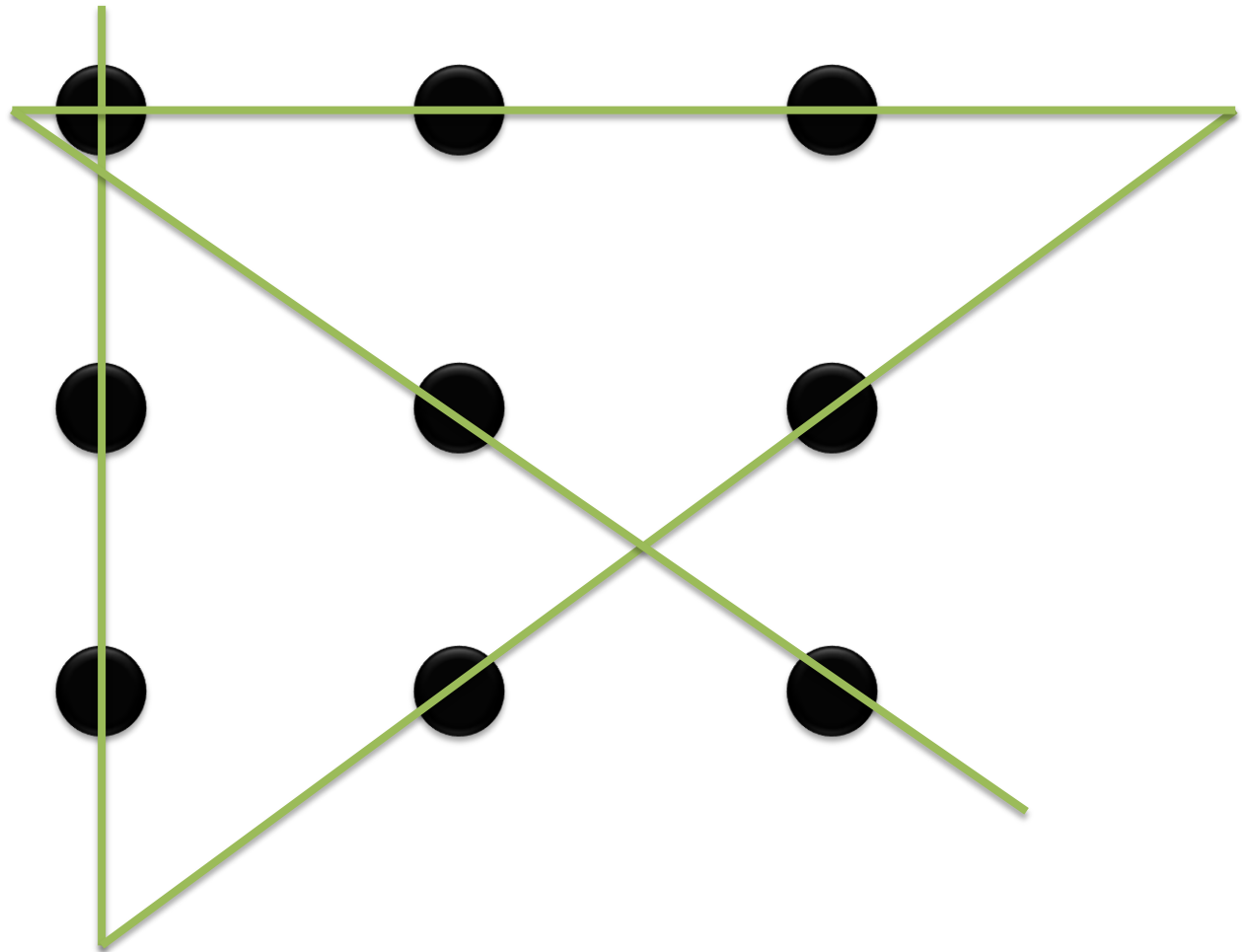
# Let's play a game!

Connect all the dots with 4 continuous straight lines.

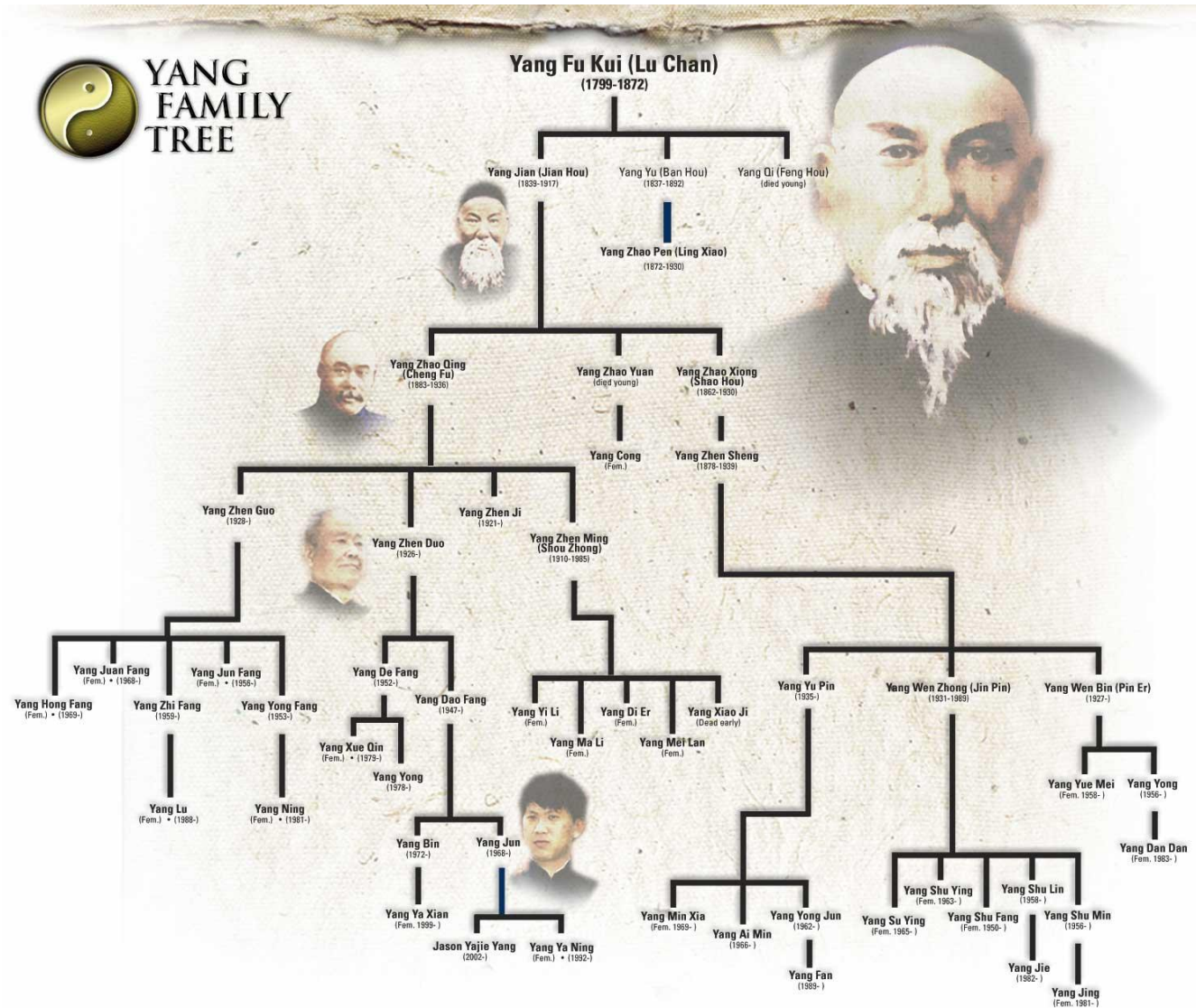


# Let's play a game!

Connect all the dots with 4 continuous straight lines.



# What is a network?



# Networks

*Social*

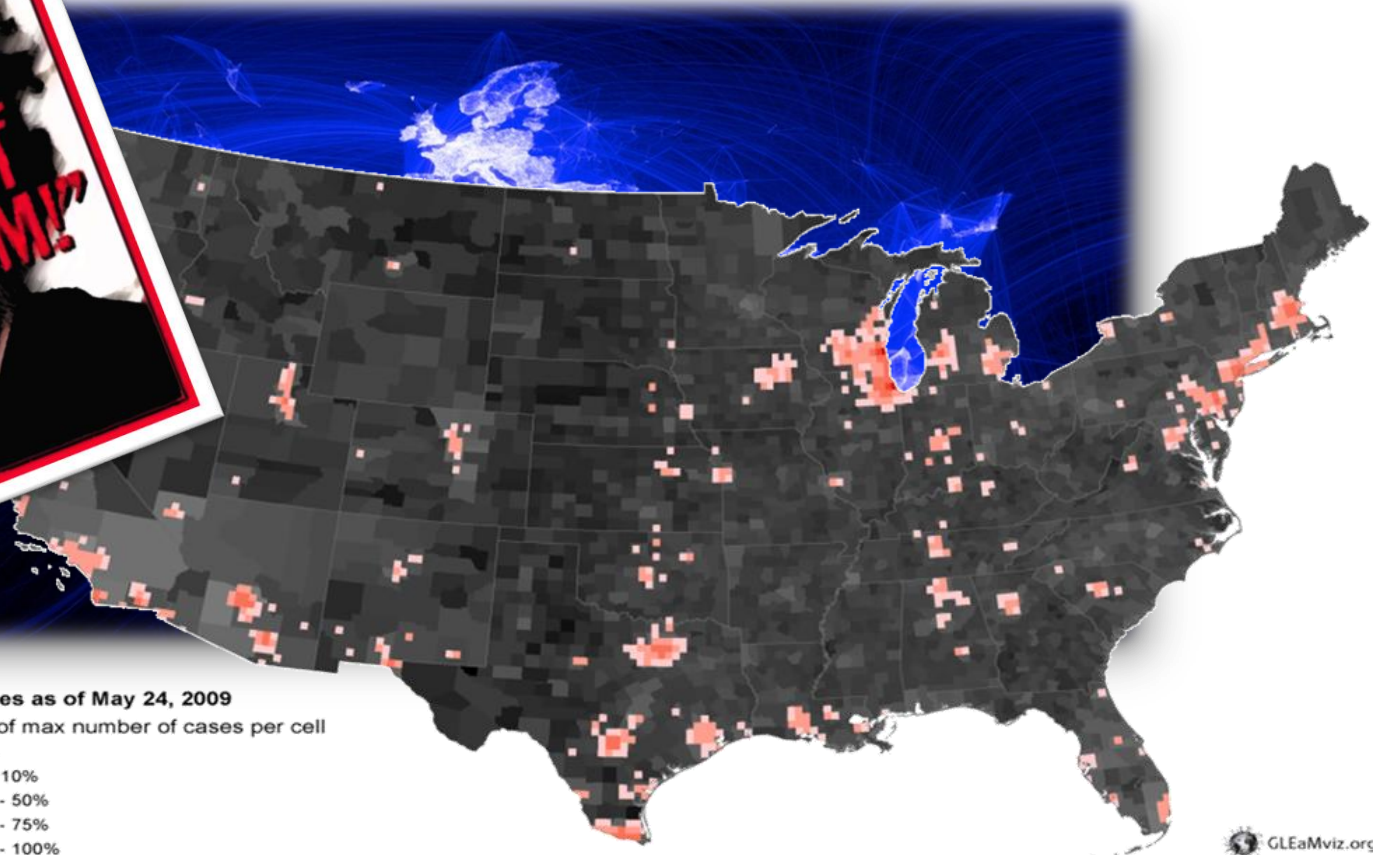


# Networks

## Social

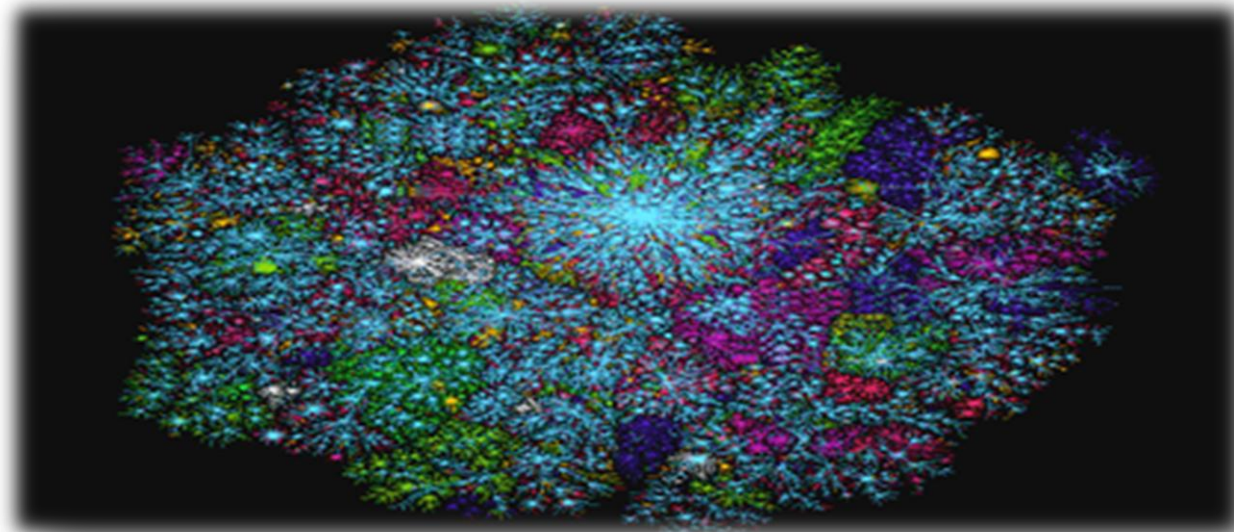


cases as of May 24, 2009  
on of max number of cases per cell  
1%  
% - 10%  
0% - 50%  
0% - 75%  
5% - 100%



# Networks

*technological*



# Networks

*technological*





# Networks

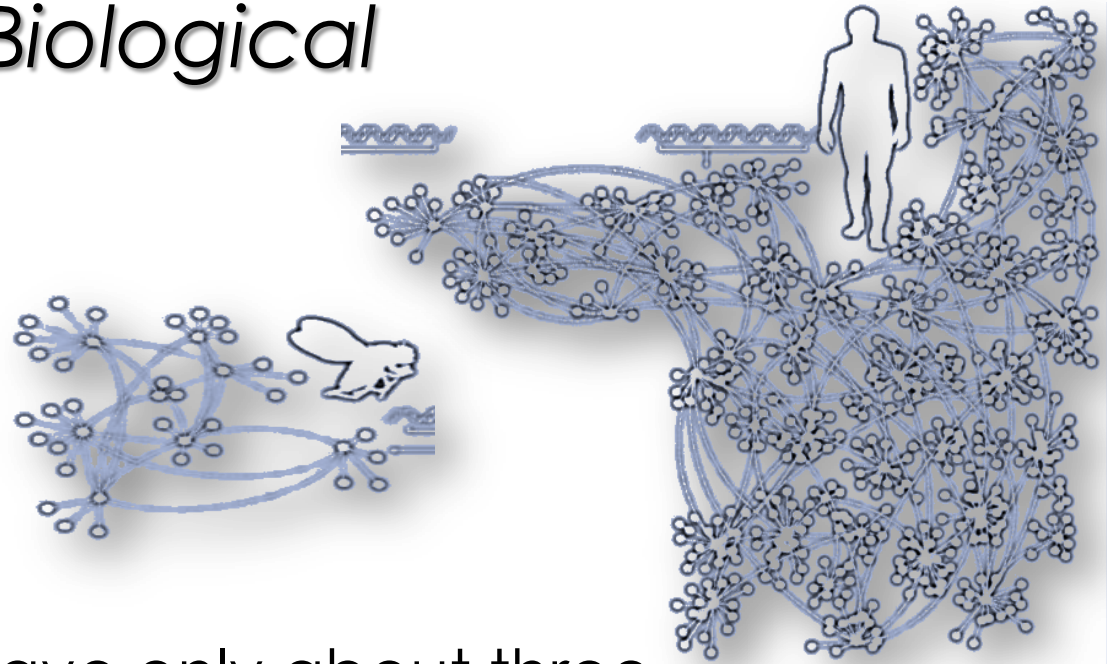
*Biological*



Humans have only about three times as many genes as the fly

# Networks

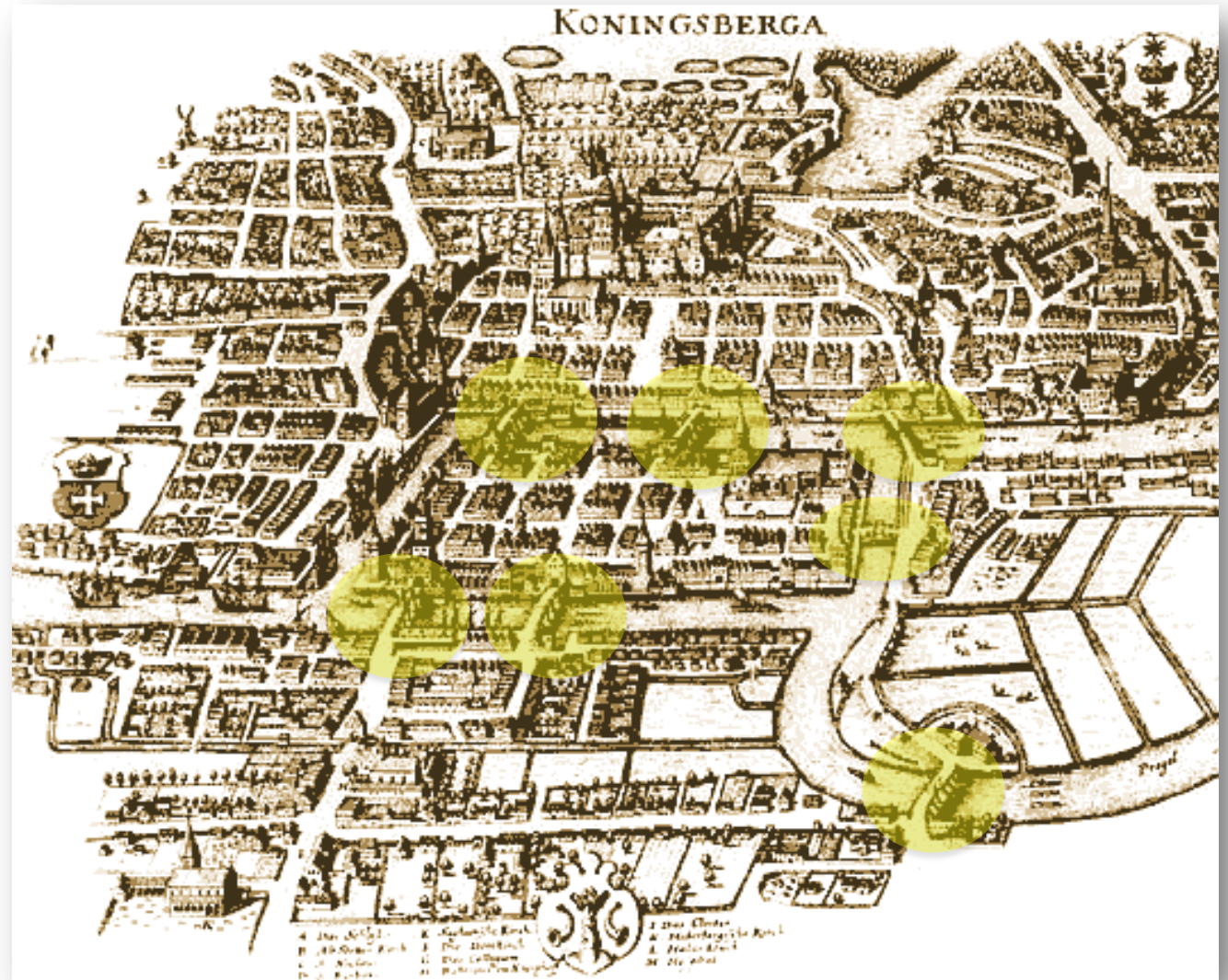
## *Biological*



Humans have only about three times as many genes as the fly

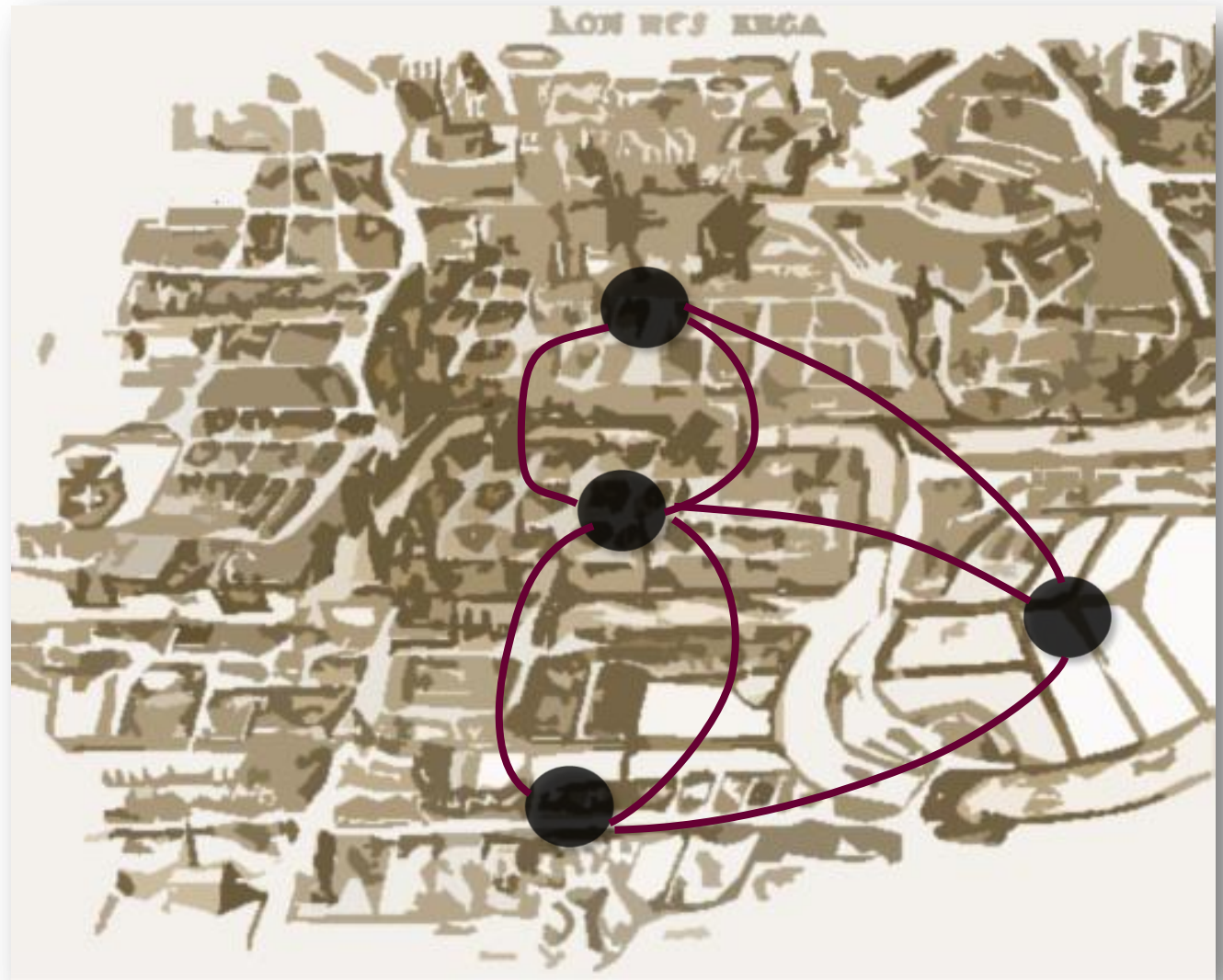
# Where it All Began – Back to 1735

Can one walk across the seven bridges and never cross the same bridge twice?



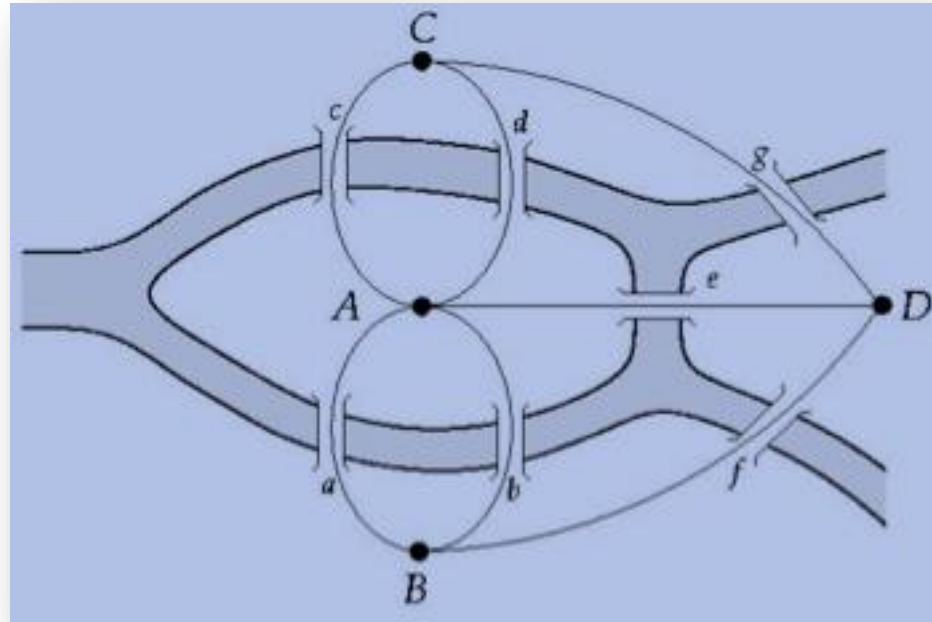
# Where it All Began – Back to 1735

Can one walk across the seven bridges and never cross the same bridge twice?



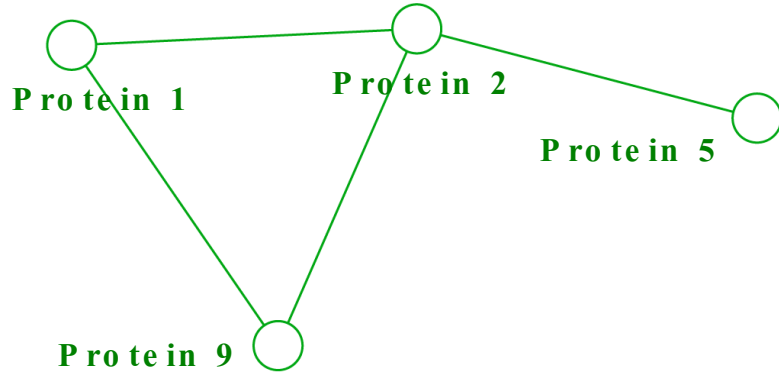
# Where it All Began – Back to 1735

Can one walk  
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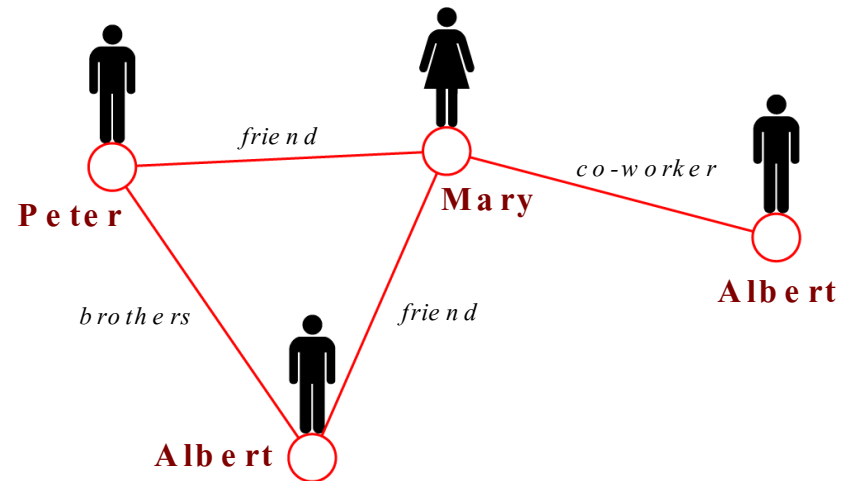
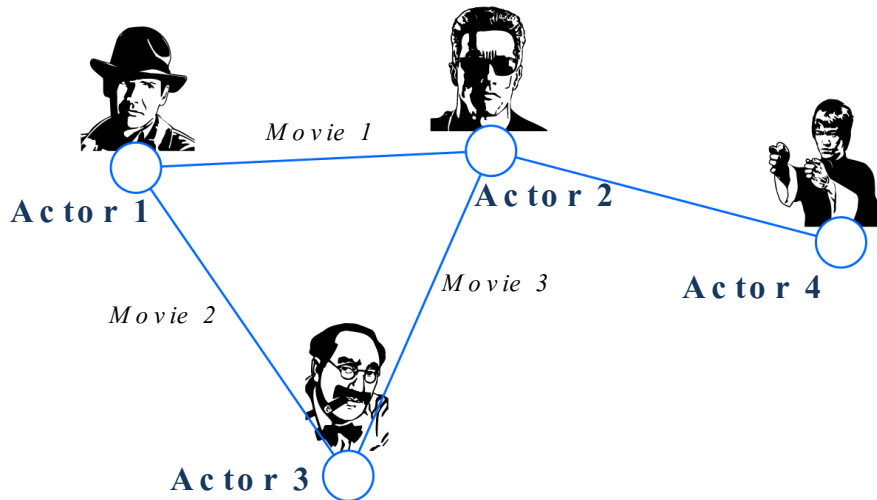


*Abstracting the problem into a  
graph allows to develop a  
universal language*

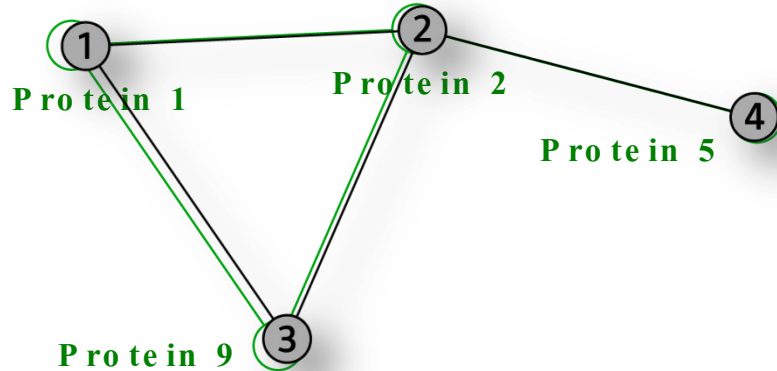
# Networks



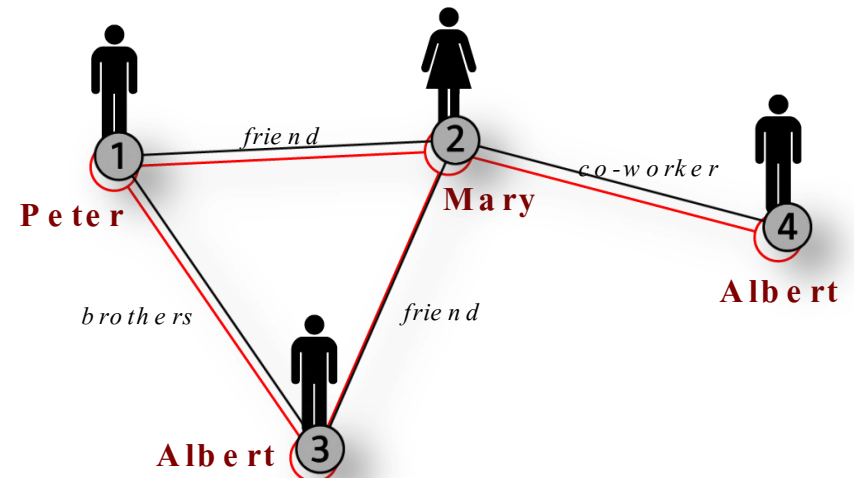
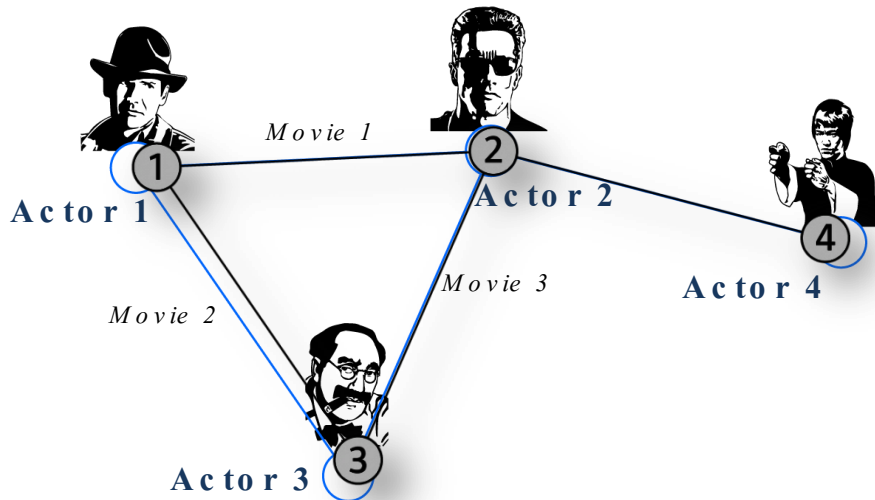
Abstracting the problem into a graph allows to develop a universal language



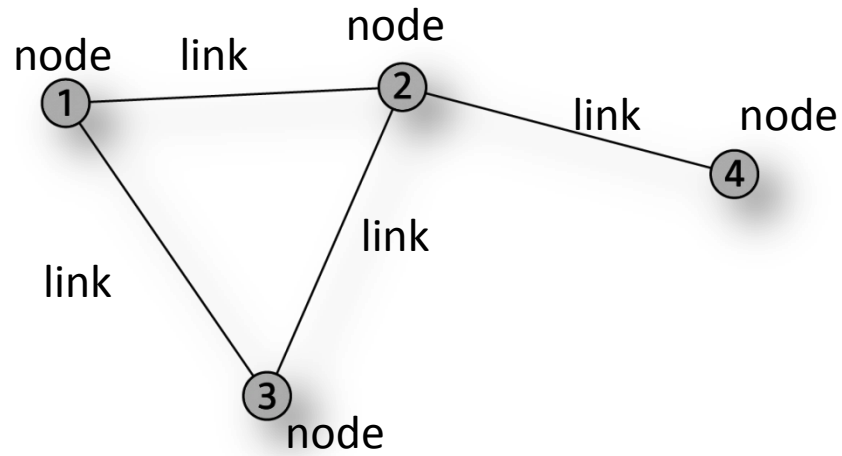
# Networks



Abstracting the problem into a graph allows to develop a universal language

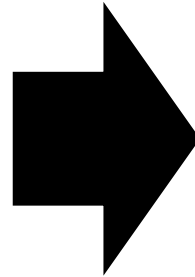


# Networks

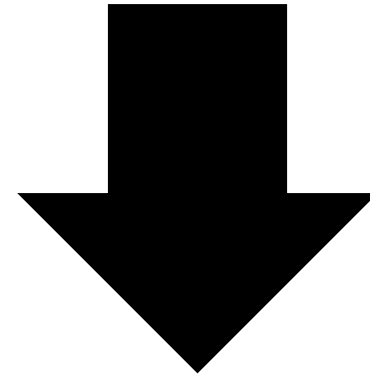




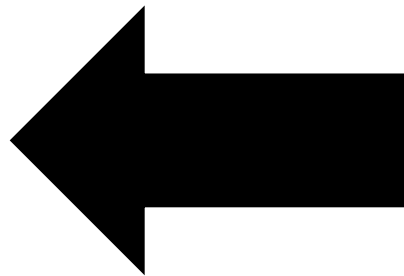
# Networks & randomness



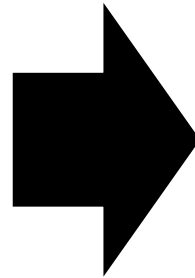
We extract a random number



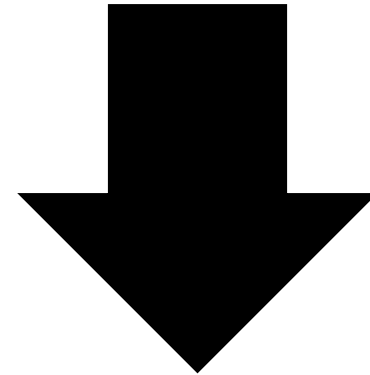
This random number represents  
a single random node



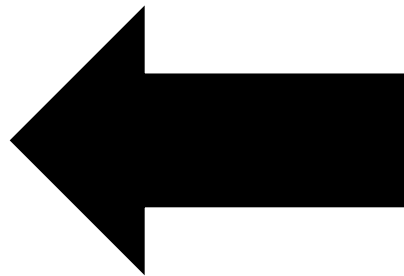
# Networks & randomness



We extract a random number AGAIN



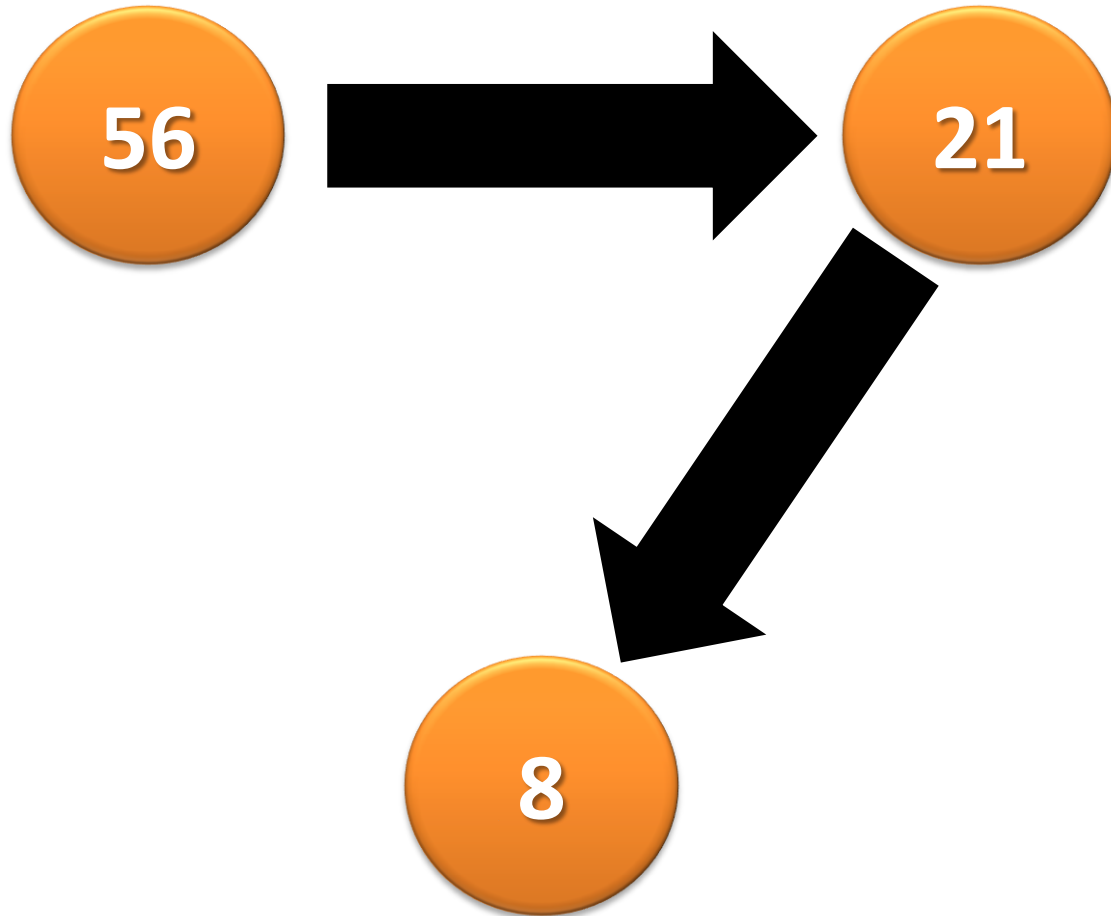
This random number represents the second single random node



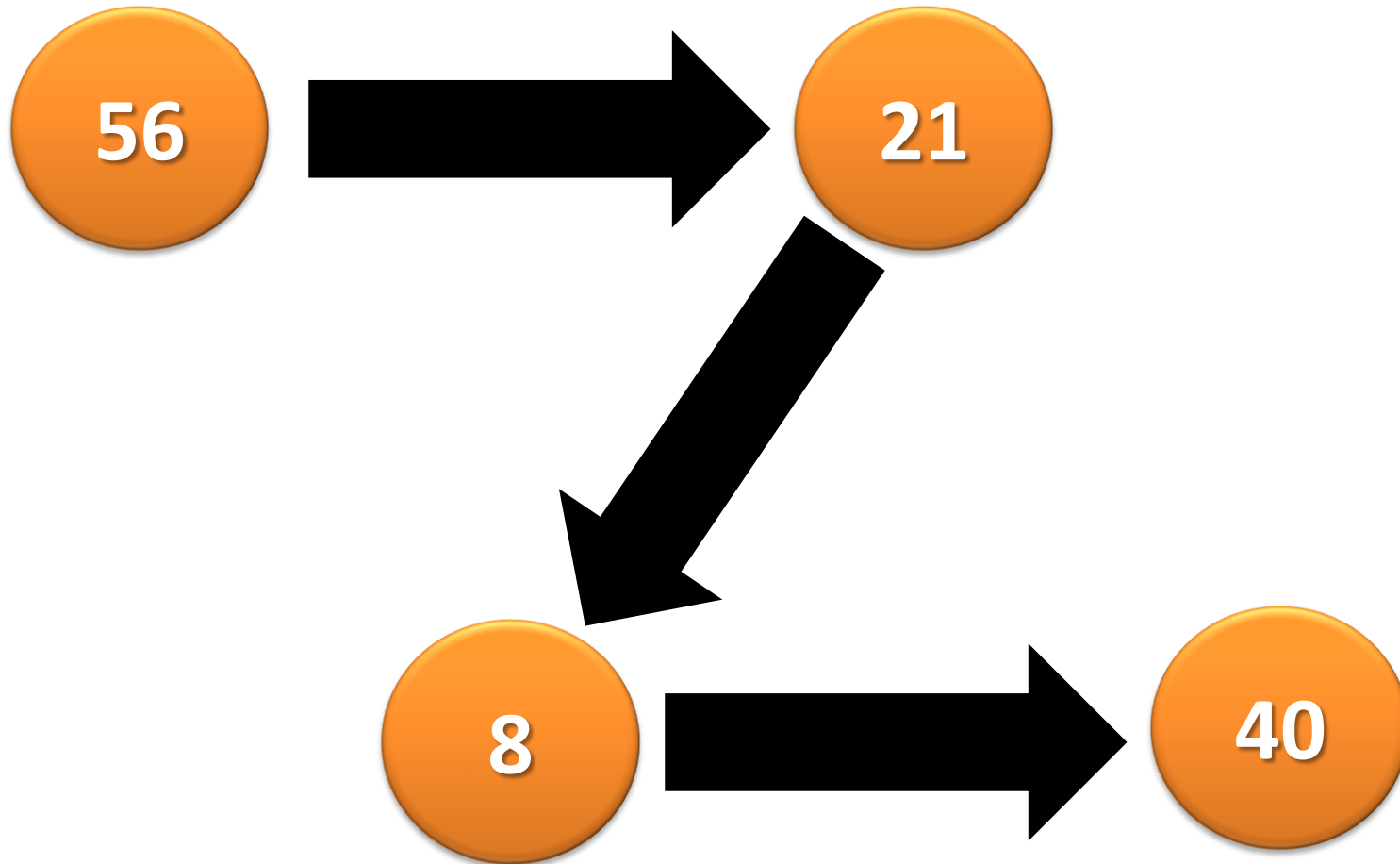
# Networks & randomness



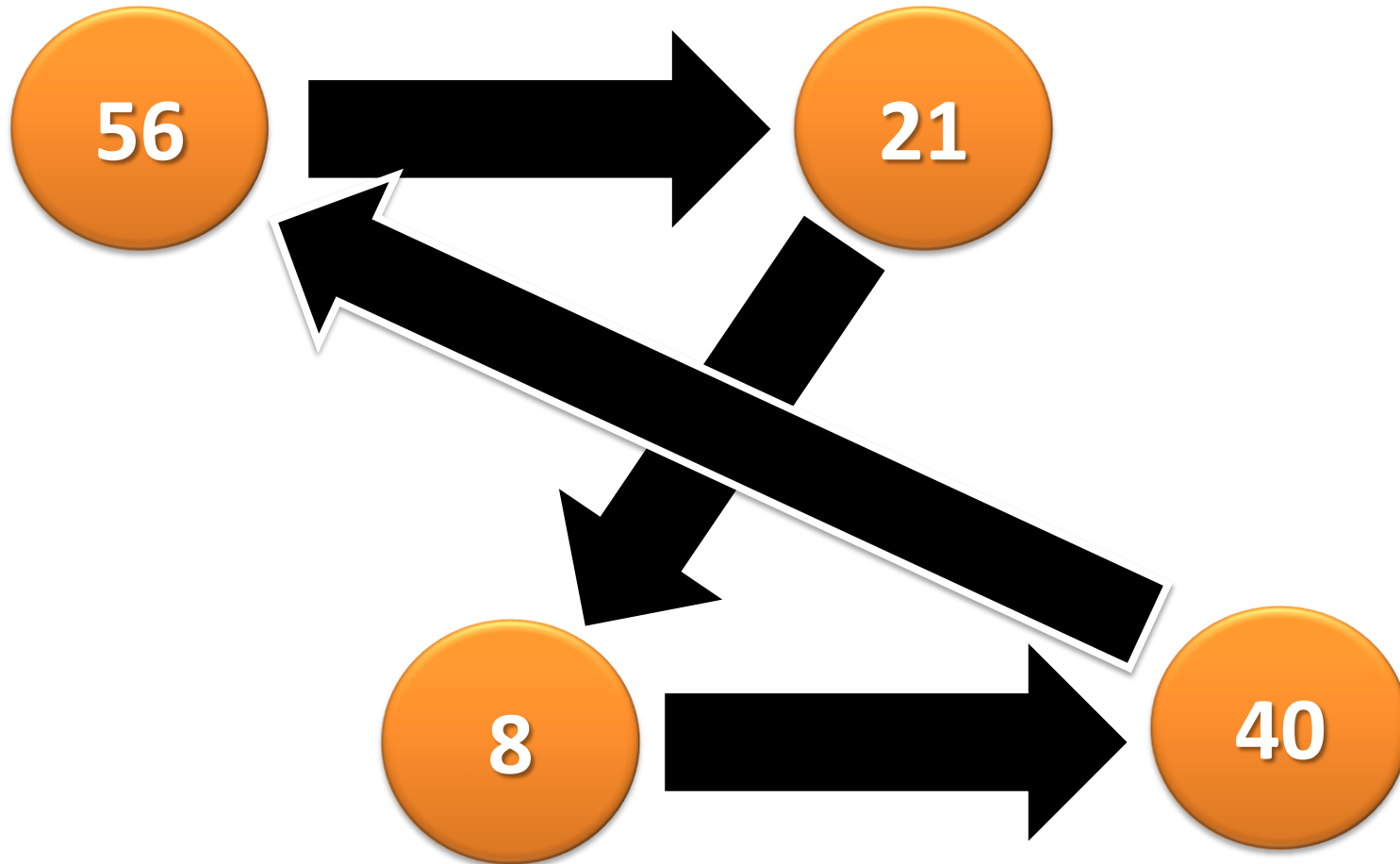
# Networks & randomness



# Networks & randomness

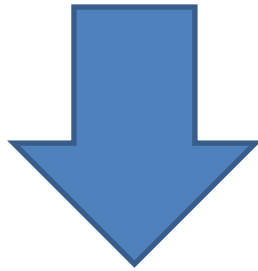


# Networks & randomness

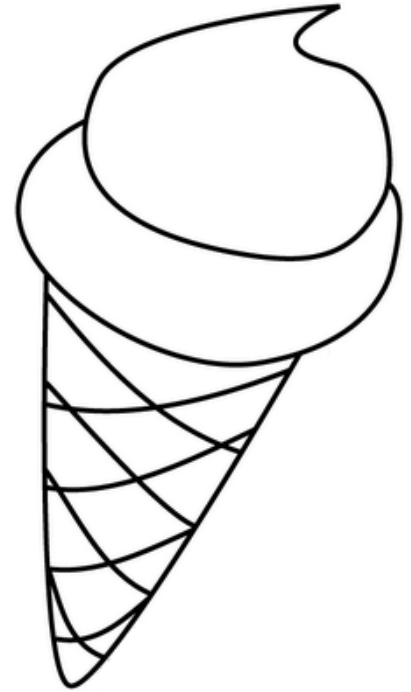


# How can we generate a network?

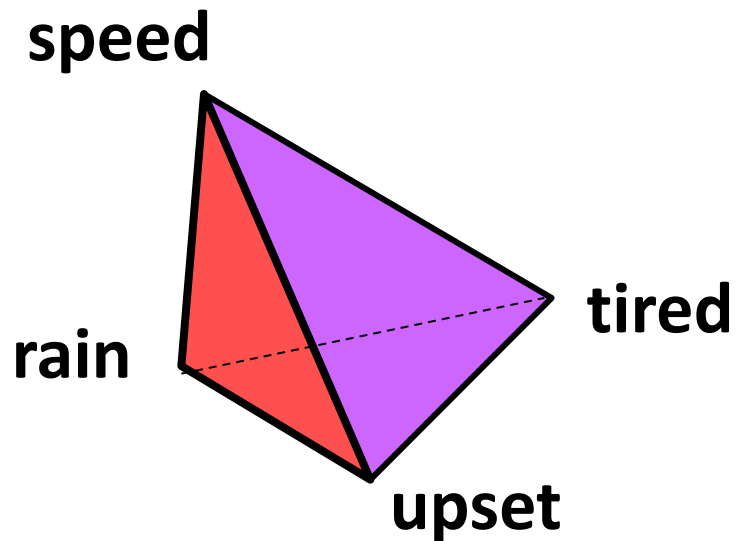
Vanilla Ice Cream  $\neq$  cold + yellow + soft + sweet + vanilla



We need a method to combine all the ingredients



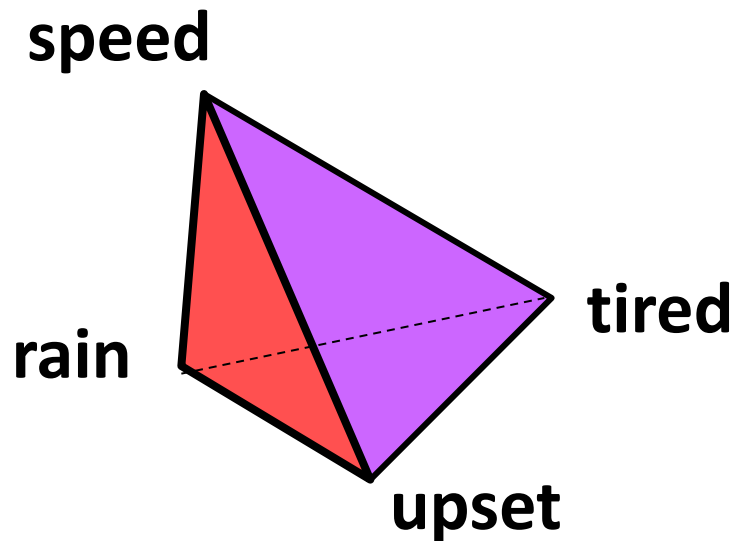
# Example: Road Accidents



The accident is  
a *whole*



# Example: Road Accidents



The accident is a ***whole***

the individual ***parts*** may not cause an accident



*The coffee break is not far ..*

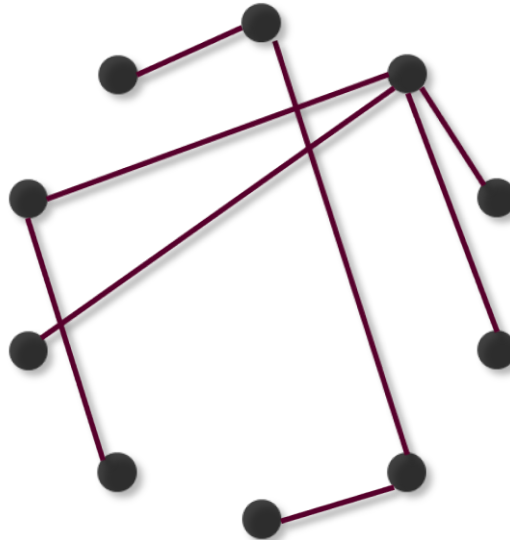
# Let's see what a network means

- Try to combine the different ingredients of a set in a way that you want.

# The Erdős-Rényi Random Graph

- *Start with  $N$  nodes*
- *Connect each pair with probability  $p$*
- *Obtain  $L$  links*

$$L_{\max} = N(N-1) / 2$$



$N = 10$   
 $p = 1/6$   
 $L = 8$

# The Erdős-Rényi Random Graph

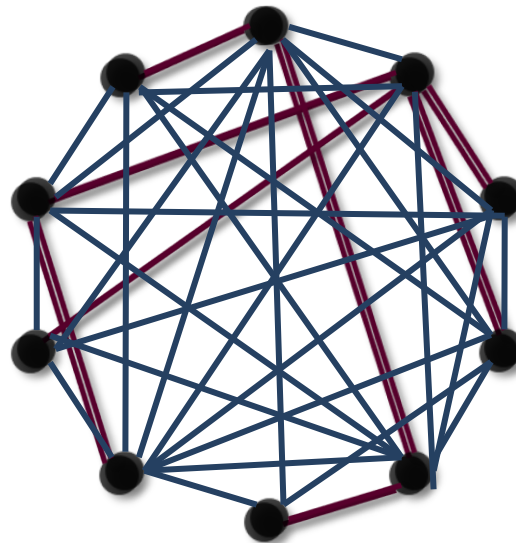
- *Start with  $N$  nodes*
- *Connect each pair with probability  $p$*
- *Obtain  $L$  links*

$$1/6 = 0.166$$

$$S = 8 / 45 = 0.177$$

$$L_{\max} = N(N-1) / 2$$

$$S = L / L_{\max}$$



$$\begin{aligned} N &= 10 \\ p &= 1/6 \\ L &= 8 \end{aligned}$$

# The Erdős-Rényi Random Graph

Density vs. Sparseness –

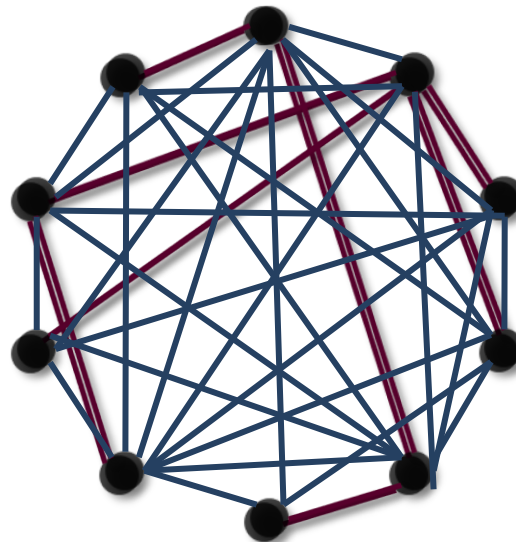
How many links are present vs. how many could there potentially be

$$1/6 = 0.166$$

$$S = 8 / 45 = 0.177$$

$$L_{\max} = N(N-1) / 2$$

$$S = L / L_{\max}$$



$$N = 10$$

$$p = 1/6$$

$$L = 8$$

# The Erdős-Rényi Random Graph

Degree -

The number of links

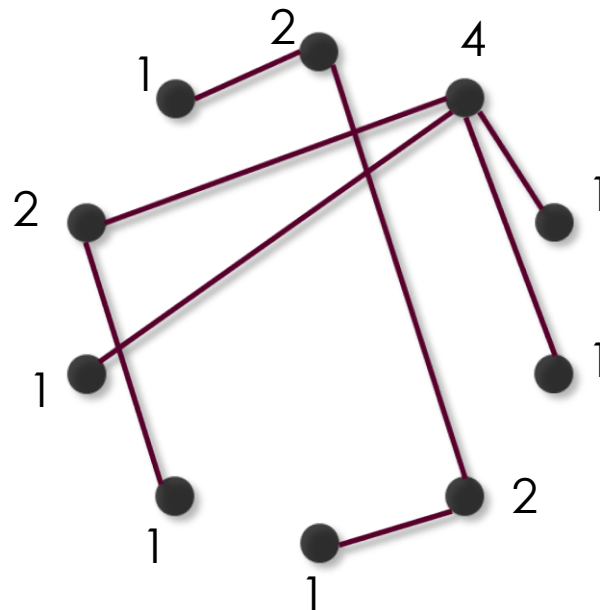
of a node

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i = \frac{2L}{N}$$

$$\langle k \rangle_{ER} = p(N - 1)$$



$$\langle k \rangle = 1.5$$

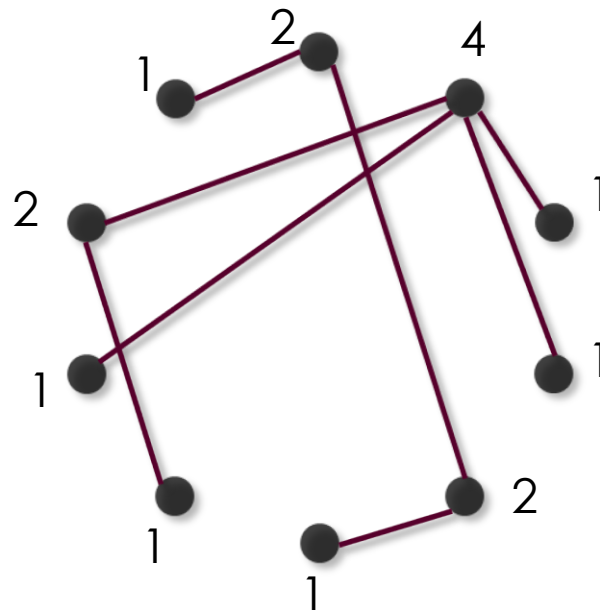
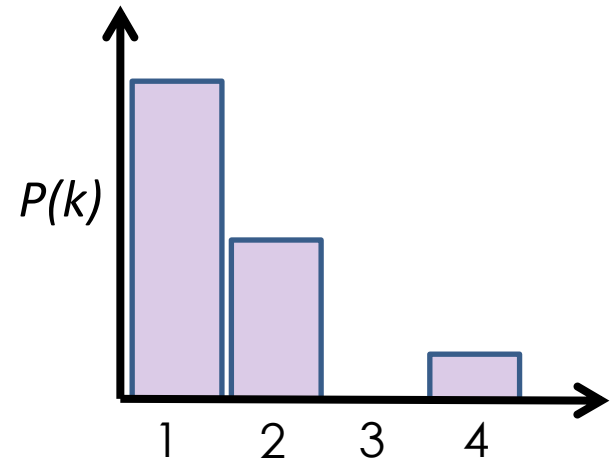


# The Erdős-Rényi Random Graph

Degree Distribution -  
The probability for a  
random node to  
have degree  $k$ .

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i = \frac{2L}{N}$$

$$\langle k \rangle_{ER} = p(N - 1)$$





# The Erdős-Rényi Random Graph

Degree Distribution -

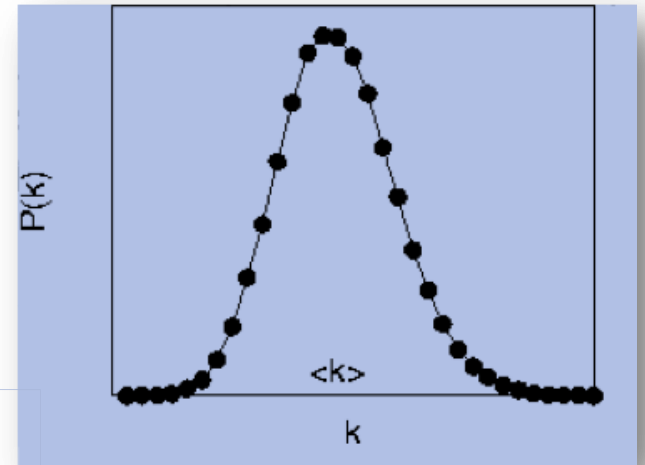
The probability for a

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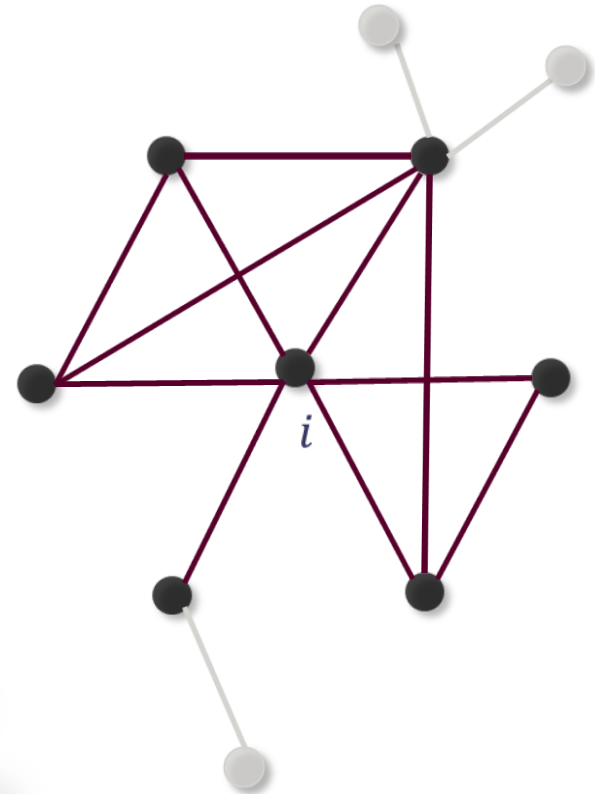
$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i = \frac{2L}{N}$$

$$\langle k \rangle_{ER} = p(N - 1)$$



# Clustering

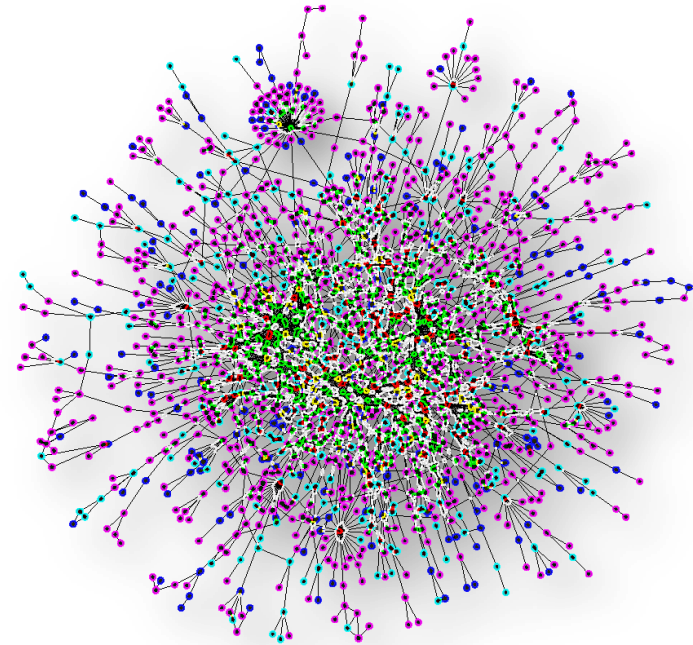
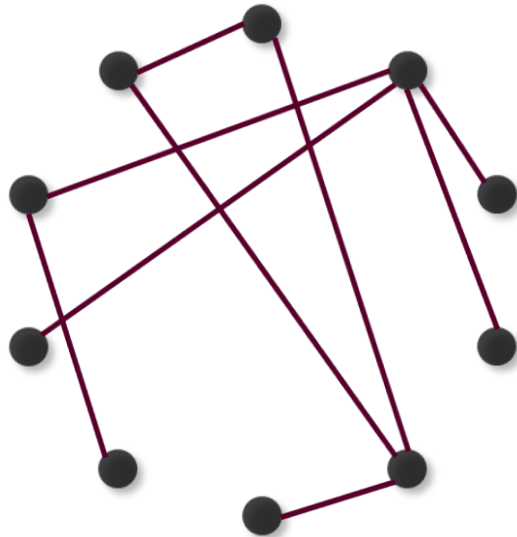
Clustering Coefficient –  
Characterizes the  
tendency to form triangles



# Types of Graphs

## Undirected

- Protein interaction networks
- Collaboration networks
- Actor co-stardom networks
- Internet

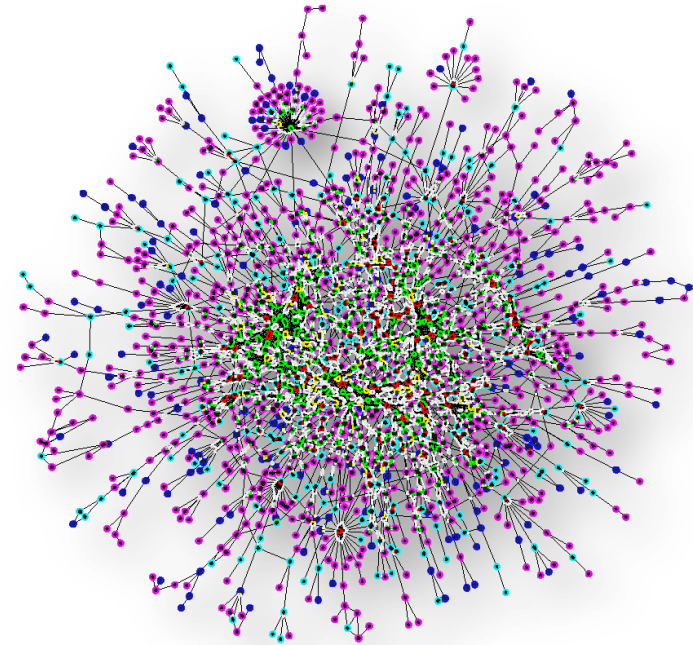
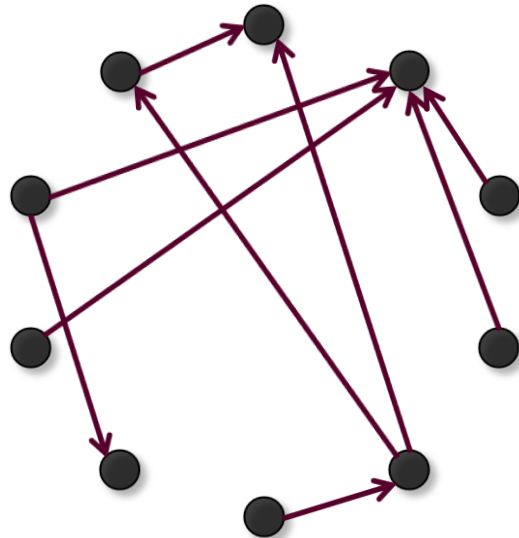


$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

# Types of Graphs

## Directed

- Metabolic
- Citation networks
- World Wide Web

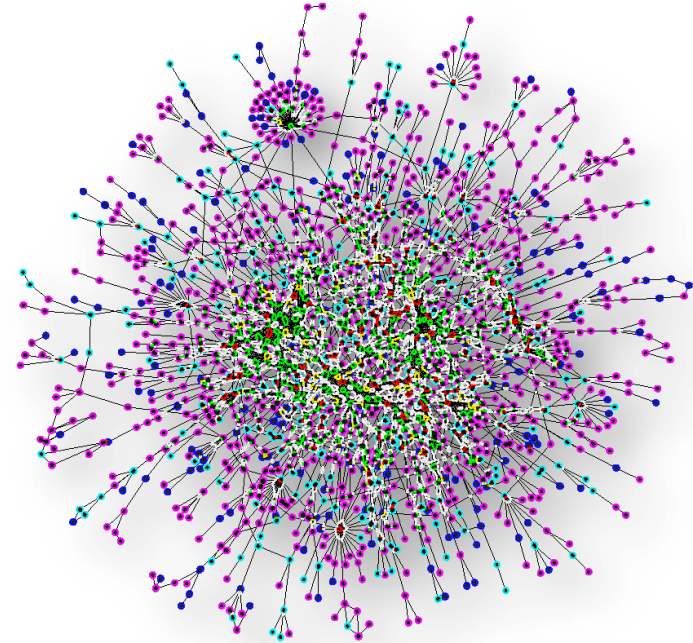
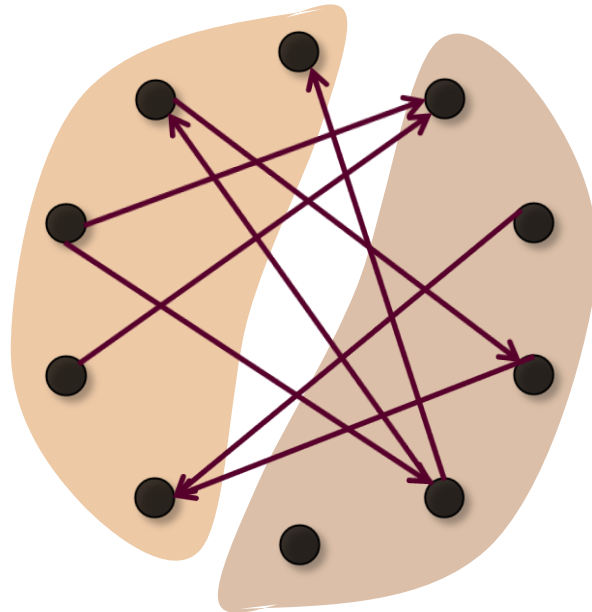


$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

# Types of Graphs

## Bipartite

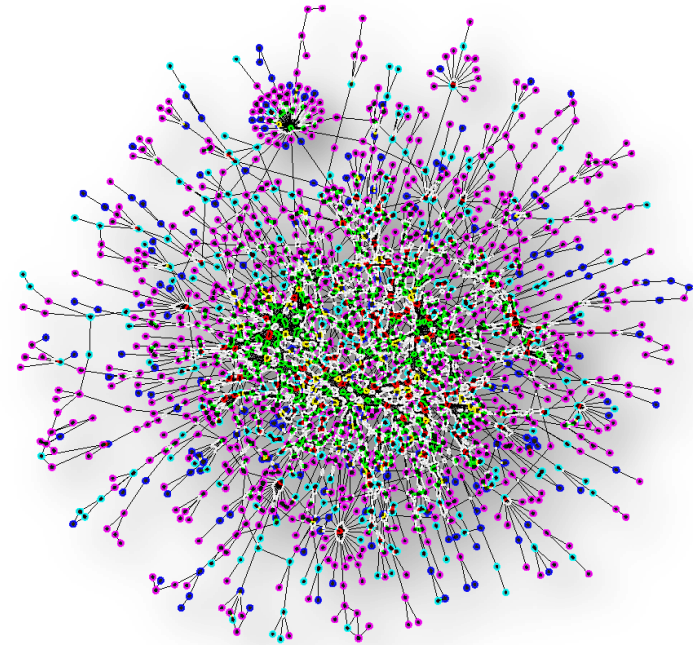
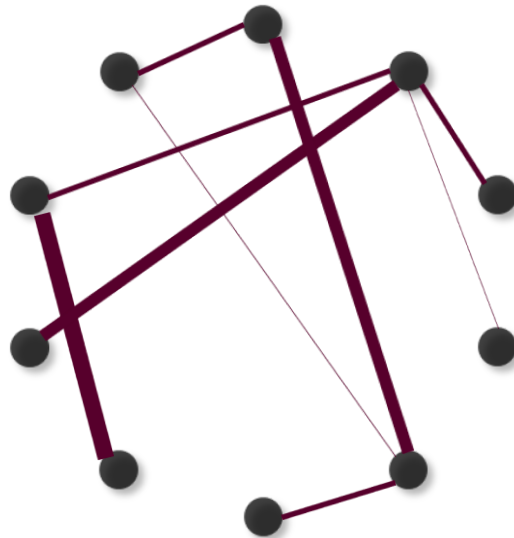
- Collaboration networks
- Actor co-stardom network
- Disease network



# Types of Graphs

## Weighted

- Metabolic networks
- Collaboration networks
- Actor co-stardom networks
- Social networks



$$A_{ij} = \begin{pmatrix} 0 & 0.8 & 0 & 0 & 0.1 & 0 \\ 0.1 & 0 & 0.3 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 0.8 & 0.9 & 0 \\ 0 & 0.3 & 0.5 & 0 & 0.3 & 0 \\ 0.2 & 0 & 0.7 & 0.2 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 0.6 & 0 \end{pmatrix}$$

# A paradigm

What kind of networks are the following ones:



Facebook



Twitter

Name	Market Price	OV / UN Valued	VE Rating	Last 12-M Return(%)	Forecast 1-Yr Return(%)	P/E Ratios	Industry	Value Level	Pivot	Risky Level
BROCADE COMM SY	\$5.53	15.3	3	0.5	4.4	10.8	NETWORKS	5.23 Q		6.08 A
DIGI INTL INC	\$9.75	7.4	3	11.5	1.5	36.3	NETWORKS	9.08 VW		12.37 S
EMULEX CORP	\$6.32	43.6	3	15.5	4.5	9.6	NETWORKS	5.76 S	6.49 Q	6.66 W
EMAGIN CORP	\$3.27	45.1	3	17.4	0.0	44.6	PERIPHERAL EQUIPMENT	3.15 VW		3.85 M
EXTREME NETWRKS	\$3.61	13.2	3	19.9	1.1	20.1	NETWORKS	3.18 M	3.68 VW	5.06 A
INTERMEC INC	\$7.98	16.7	3	8.6	0.3	72.6	PERIPHERAL EQUIPMENT	5.04 M		8.08 W
INFINERA CORP	\$5.79	26.9	3	15.4	3.0	N/A	NETWORKS	6.01 VW		6.87 S
KEY TRONIC	\$9.87	21.3	3	105.6	2.5	7.5	PERIPHERAL EQUIPMENT	7.49 Q	9.94 VW	10.37 M
LOGITECH INTL	\$7.52	31.7	3	3.2	2.8	14.0	PERIPHERAL EQUIPMENT	6.12 VW		9.87 M
MITEK SYSTEMS	\$3.45	106.8	3	58.4	0.1	N/A	OPTICAL RECOGNITION	3.01 A	3.24 A	5.57 S
NOVATEL WIRELES	\$1.31	18.5	2	56.5	9.6	N/A	PERIPHERAL EQUIPMENT	1.09 VW		3.67 Q
PLANAR SYSTEMS	\$1.19	49.5	2	40.2	9.1	N/A	PERIPHERAL EQUIPMENT	1.05 S	1.18 VW	1.78 Q
OLOGIC CORP	\$8.83	49.1	3	41.3	4.1	11.4	NETWORKS	7.57 A	8.60 VW	12.02 Q
SIERRA WIRELESS	\$7.95	26.0	3	14.9	3.1	12.0	PERIPHERAL EQUIPMENT	7.53 A	8.14 Q	9.17 M
TRANSACT TECH	\$7.43	9.6	3	12.2	1.1	16.2	PERIPHERAL EQUIPMENT	6.74 S	7.53 M	7.72 W

stocks



Family



internet

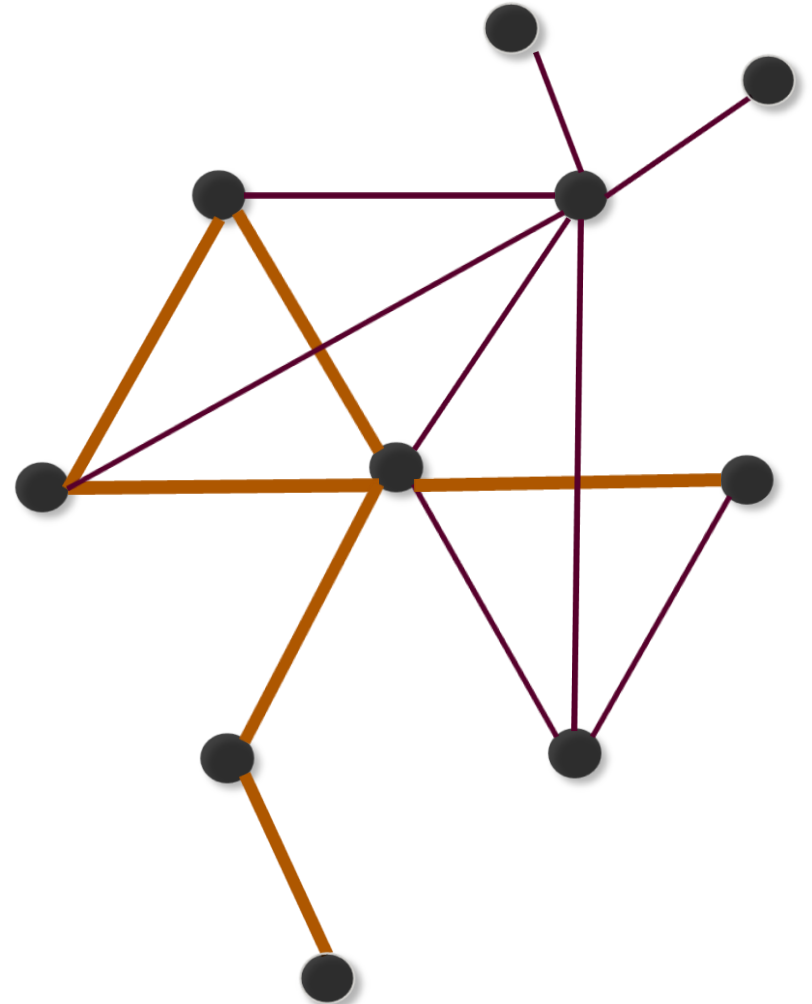


Transportation

# The metric of paths

Network Distance –  
the minimum number of edges  
between a pair of nodes

$$D_{ij} = 3$$

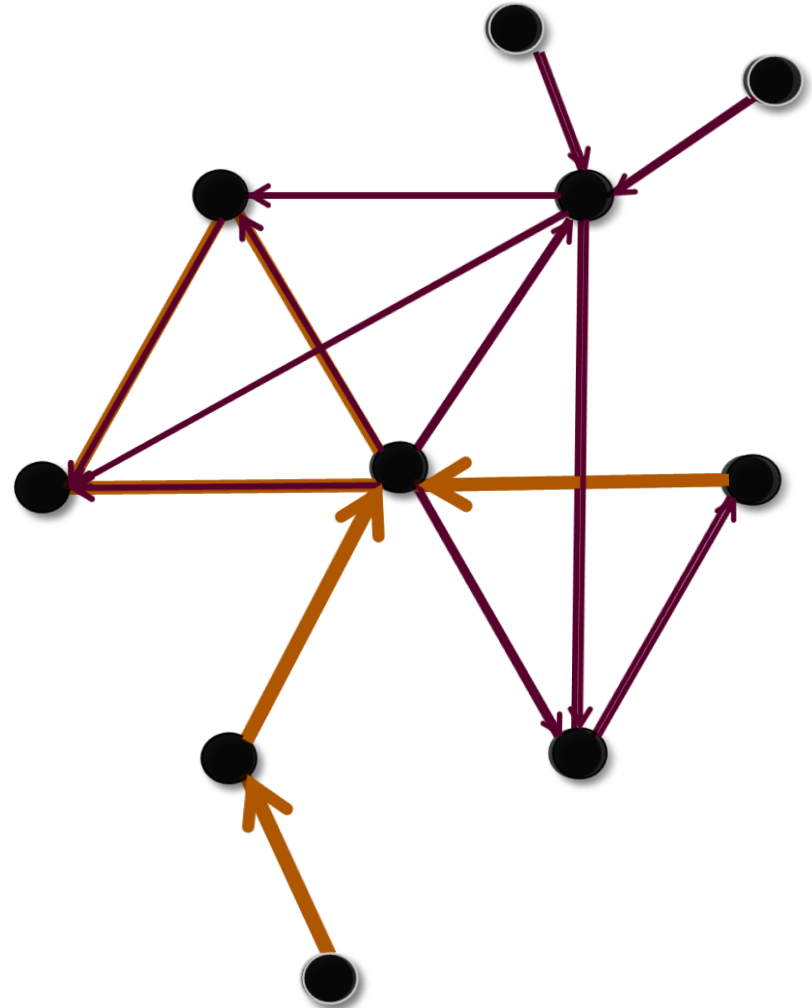




# The metric of paths

Network Distance –  
the minimum number of edges  
between a pair of nodes

$$D_{ij} = 4$$



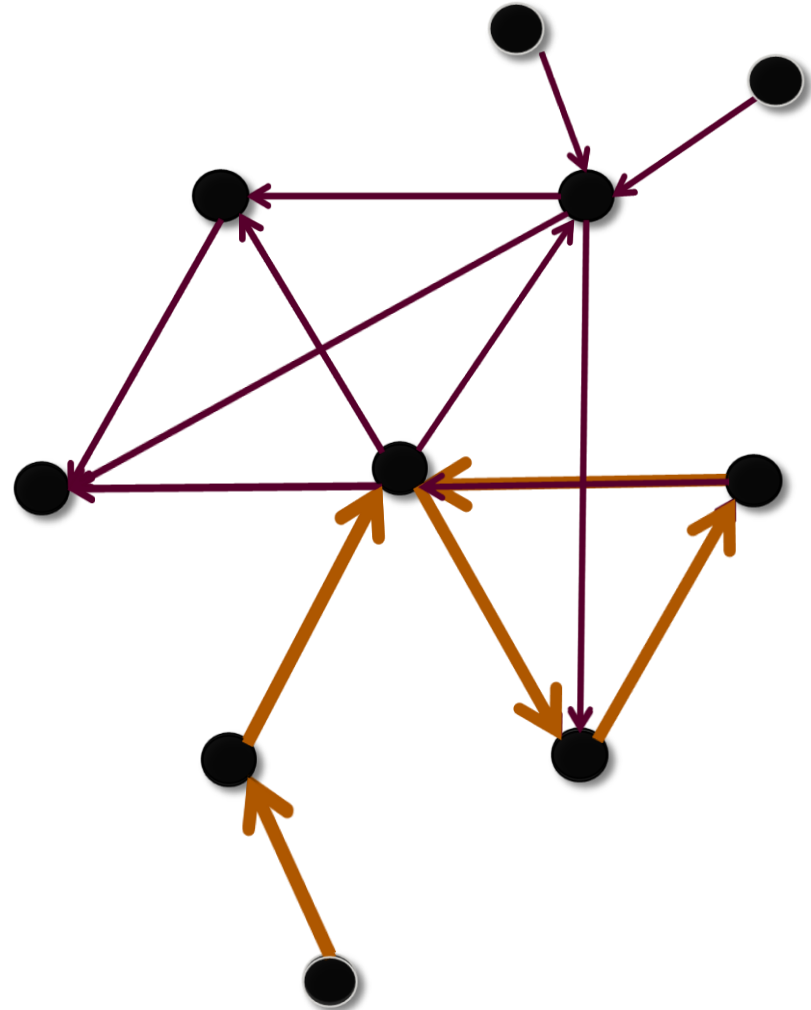
# The metric of paths

Network Distance –  
the minimum number of edges  
between a pair of nodes

$$D_{ji} = \infty$$



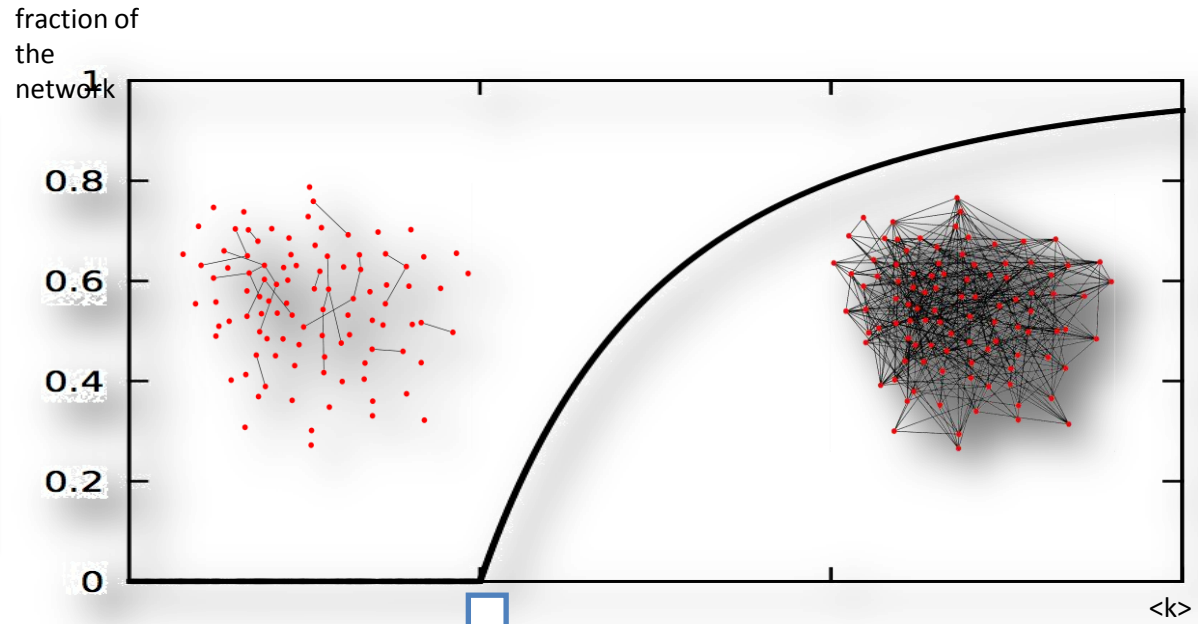
$$D_{ij} = 4$$



# Giant Component

Component –

A group of nodes that can be reached by finite paths from one another



$$\langle k \rangle_{\text{crit.}} = 1$$

# Giant Component

Radius –

Average path length

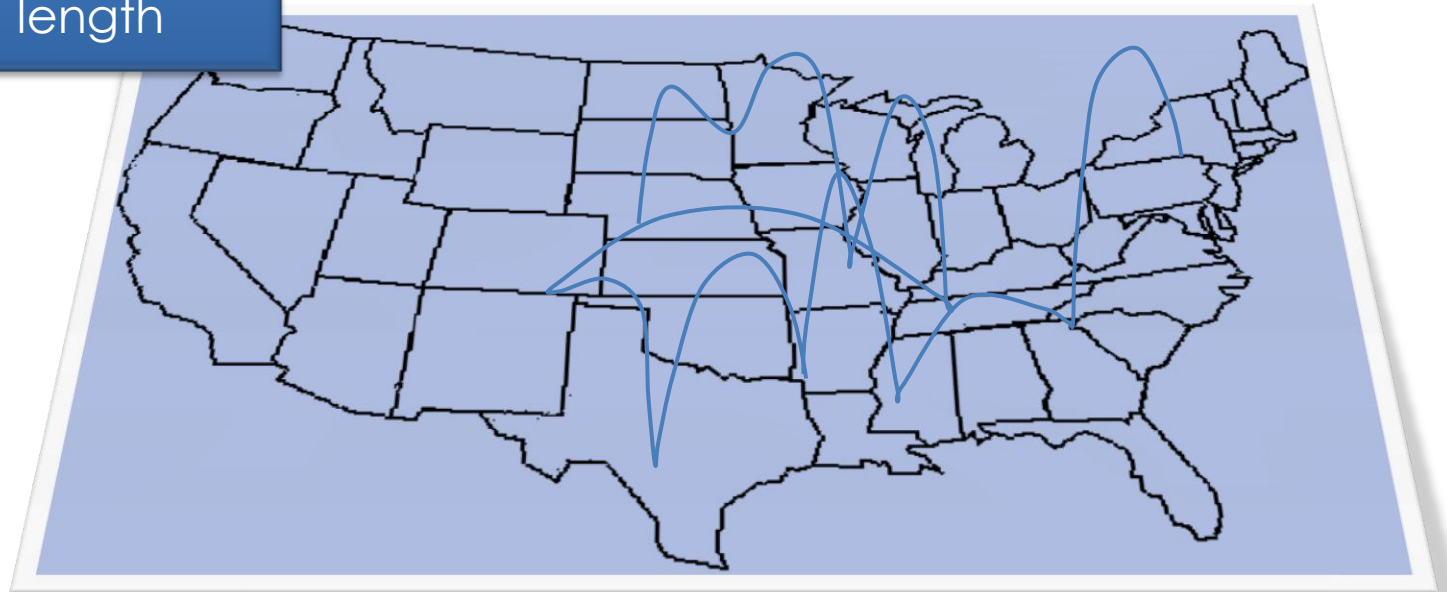
Diameter –

Maximum path length

HOMEWORK 1 :

Search for ..

“Milgram’s Experiment -  
Six Degrees of Separation”



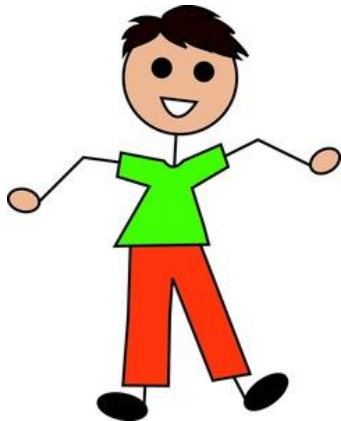
# A Paradigm



# A Paradigm



# A Paradigm



# A Paradigm

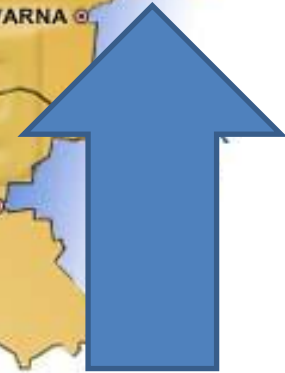
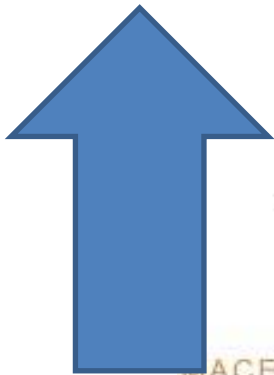




# A Paradigm



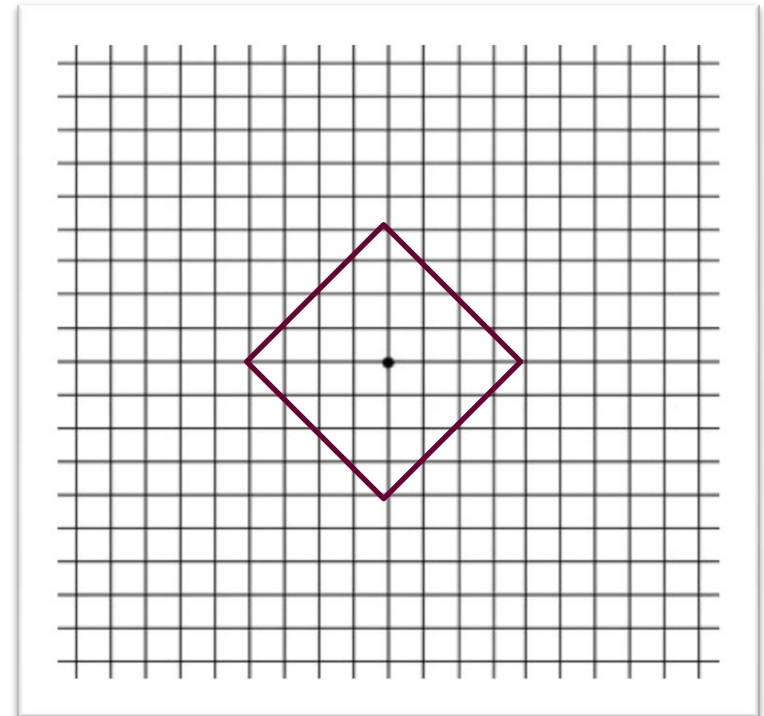
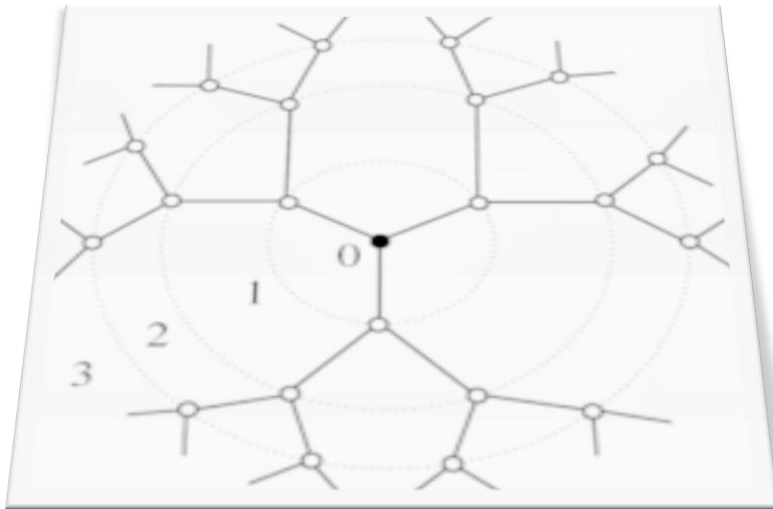
# A Paradigm



# The exploding volume of networks

The secret behind the small world effect –  
Looking at the network volume

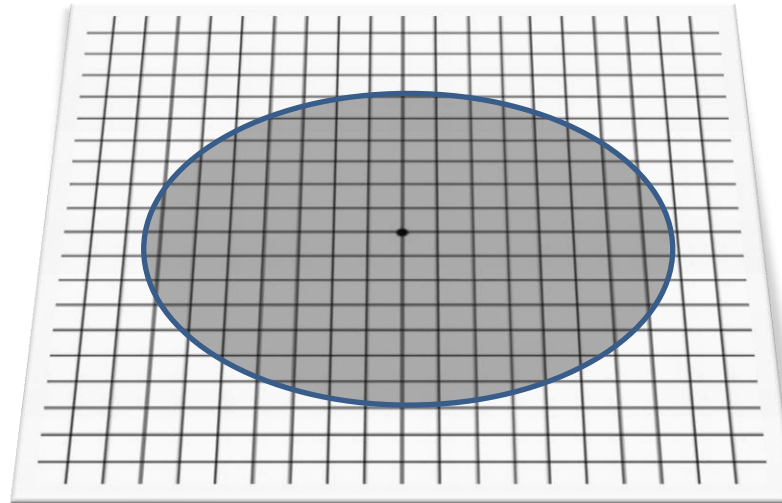
$$S(d) = 4d$$



# The exploding volume of networks

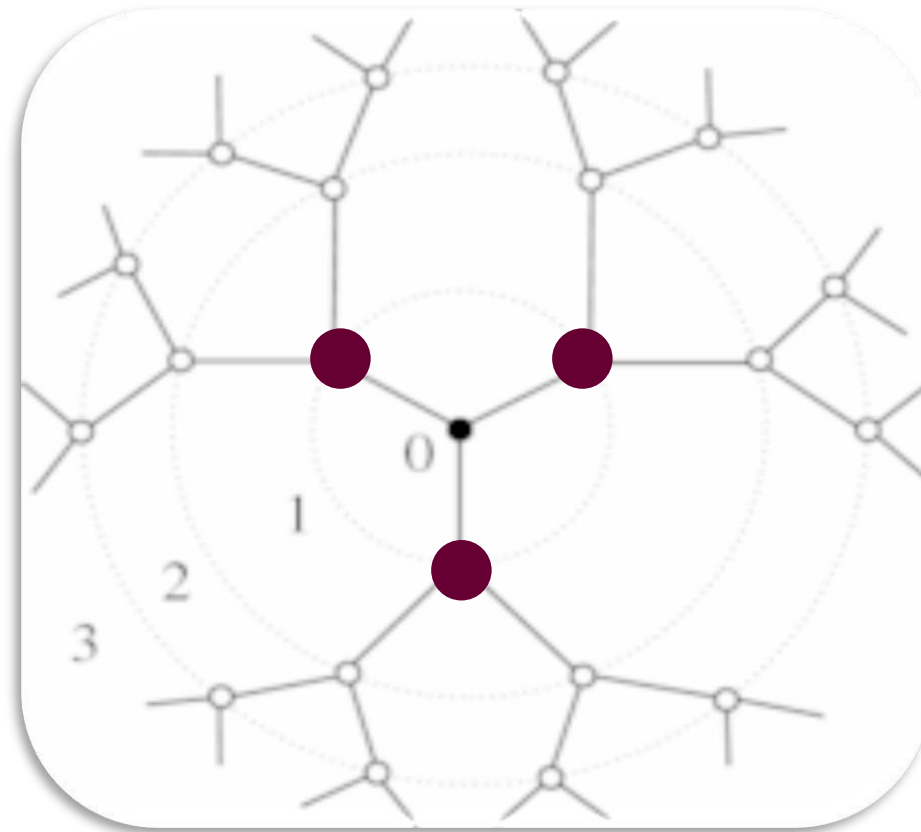
The secret behind the small world effect –  
Looking at the network volume

$$N(d) = \sum_{x=1}^d 4x = 2d(d+1) \sim d^2$$



# The exploding volume of networks

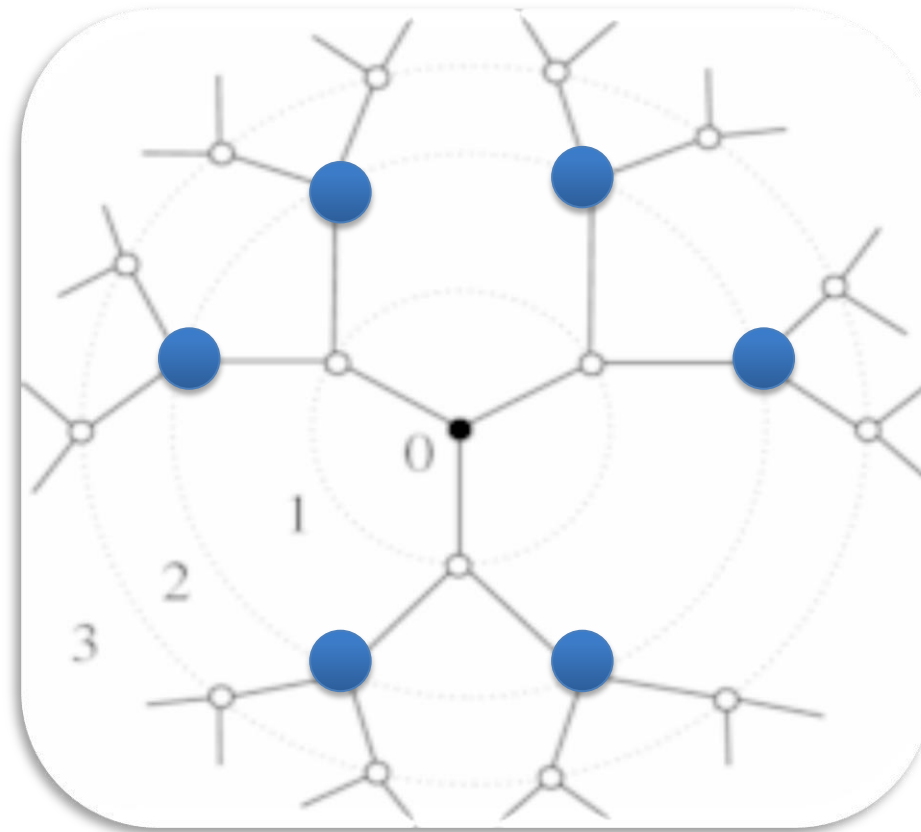
First Neighborhood



● x 3

# The exploding volume of networks

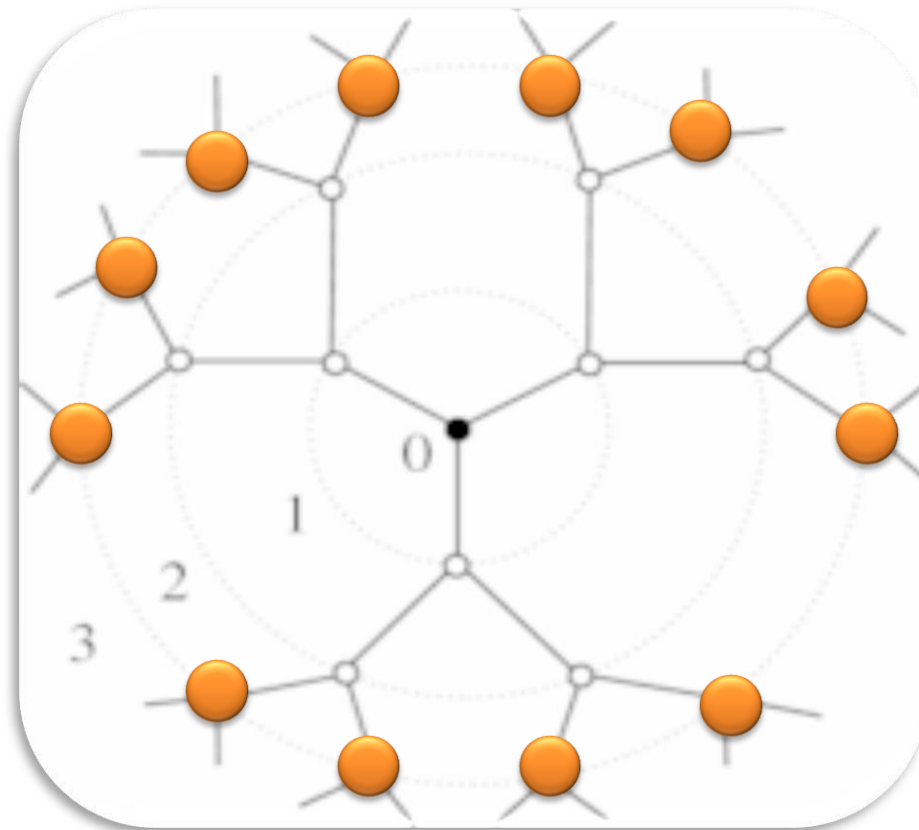
Second Neighborhood



● x 6

# The exploding volume of networks

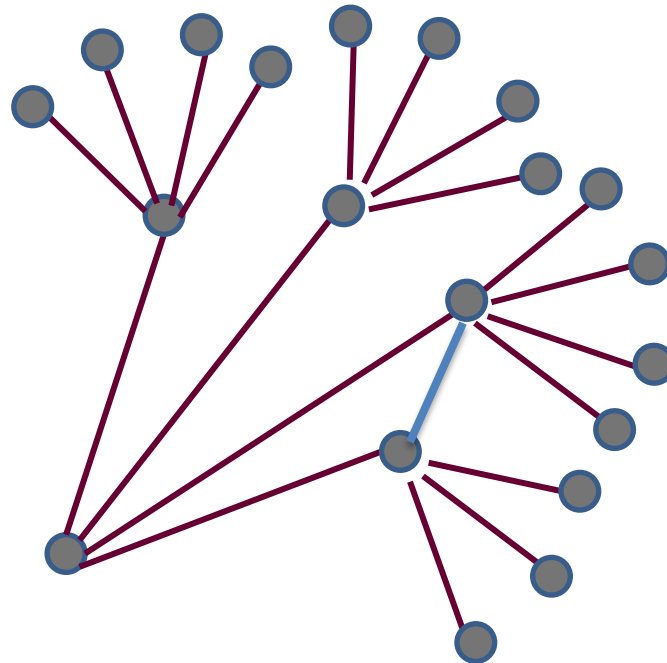
Third Neighborhood



● x 12

# TREES ?

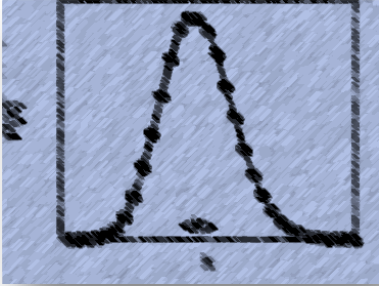
Random Graphs are NOT trees.



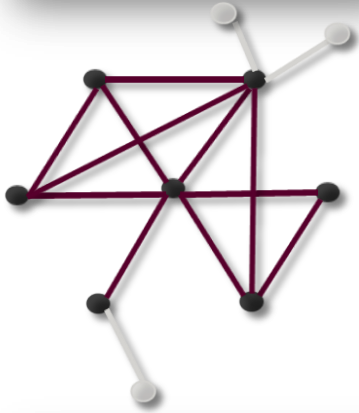
***Some of your  
neighbor's  
neighbors are  
also your own***



# The Erdős-Rényi Random Graph



Poisson



Clustering



Small world

radius scales logarithmically with volume

# Exercise 1

## *Random Network*

Create a network with  $N$  nodes.

Use  $N=10000$  and  $N=100000$  for a Random distribution of connections.

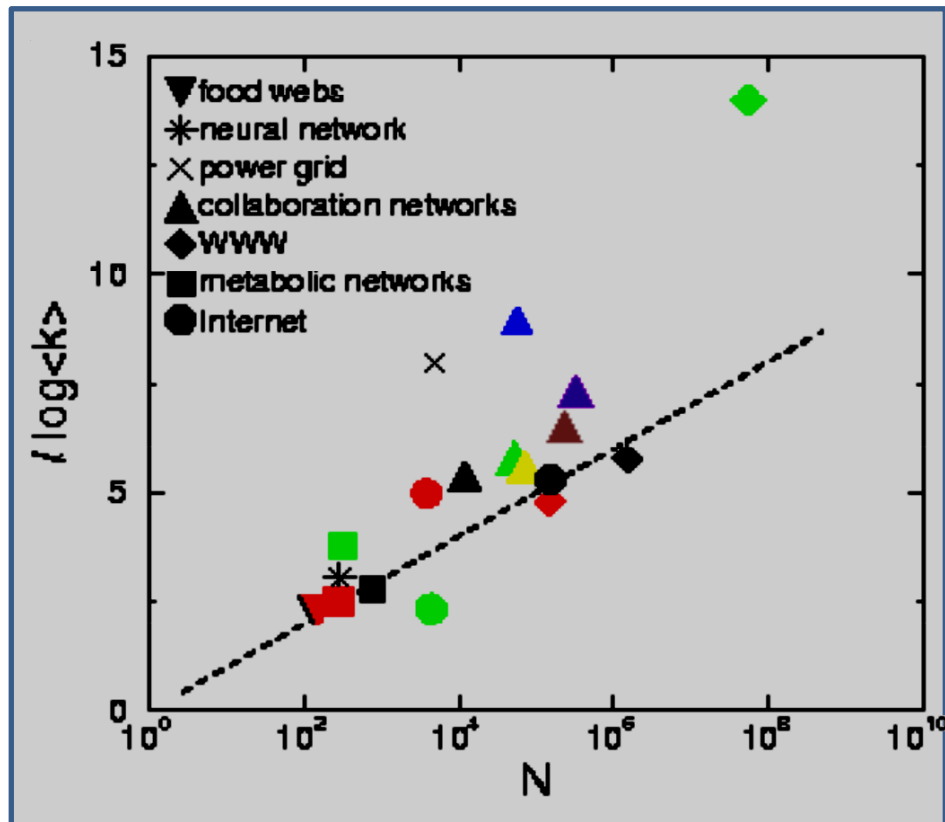
Use the rule that for every possible connection between two (2) nodes there is a probability of  $1/6$ .

Find the  $k$  of every node, where  $k$  is its number of connections.

Make the distribution of  $P(k)$  and plot  $P(k)$  vs  $k$  on a graph.

The data will be the average of 100 runs.

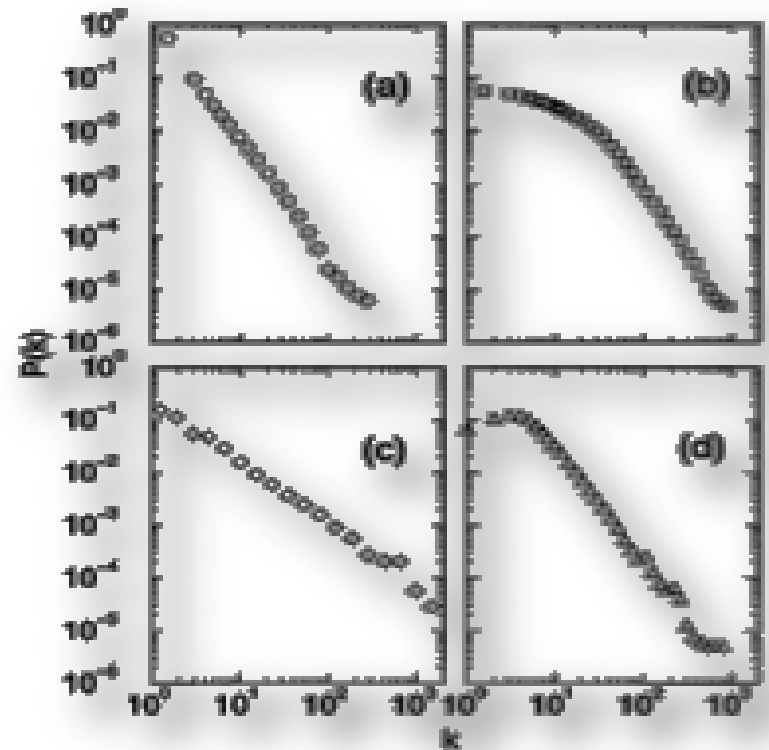
# The Erdős-Rényi Model can be used in Real Networks ?



# The Erdős-Rényi Model and Real Networks

It is the reference model – a standard candle

It will help us calculate many quantities, that can then be compared to the real data, understanding to what degree is a particular property the result of some random process.



Which is the type of Real Networks in Nature ?

# Scale-free Networks

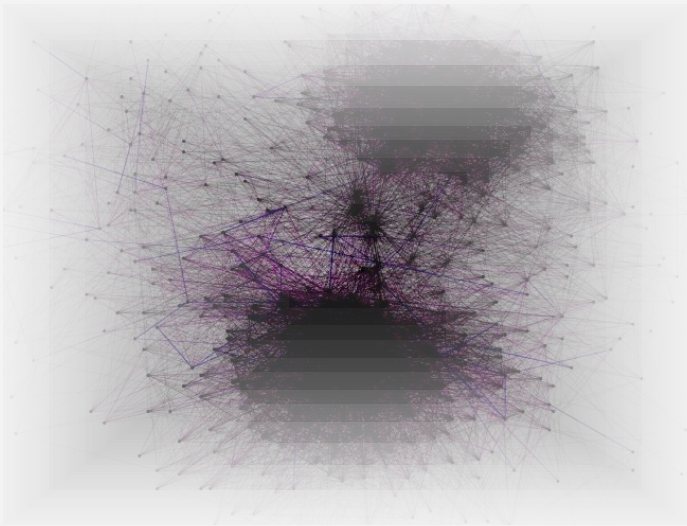
**CLUSTERING**

**RANDOMNESS**

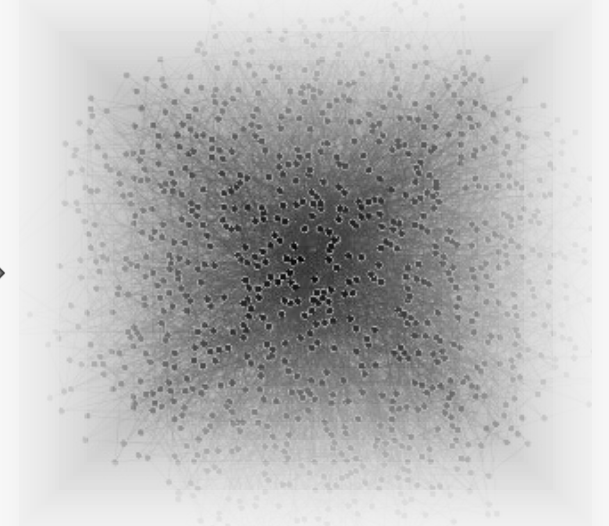


# Scale-free Networks

Where should we place the social network?



Clustered



Random

# Scale-free Networks

Could a network which is so strongly locally structured be at the same time a small world?



# Scale-free Networks

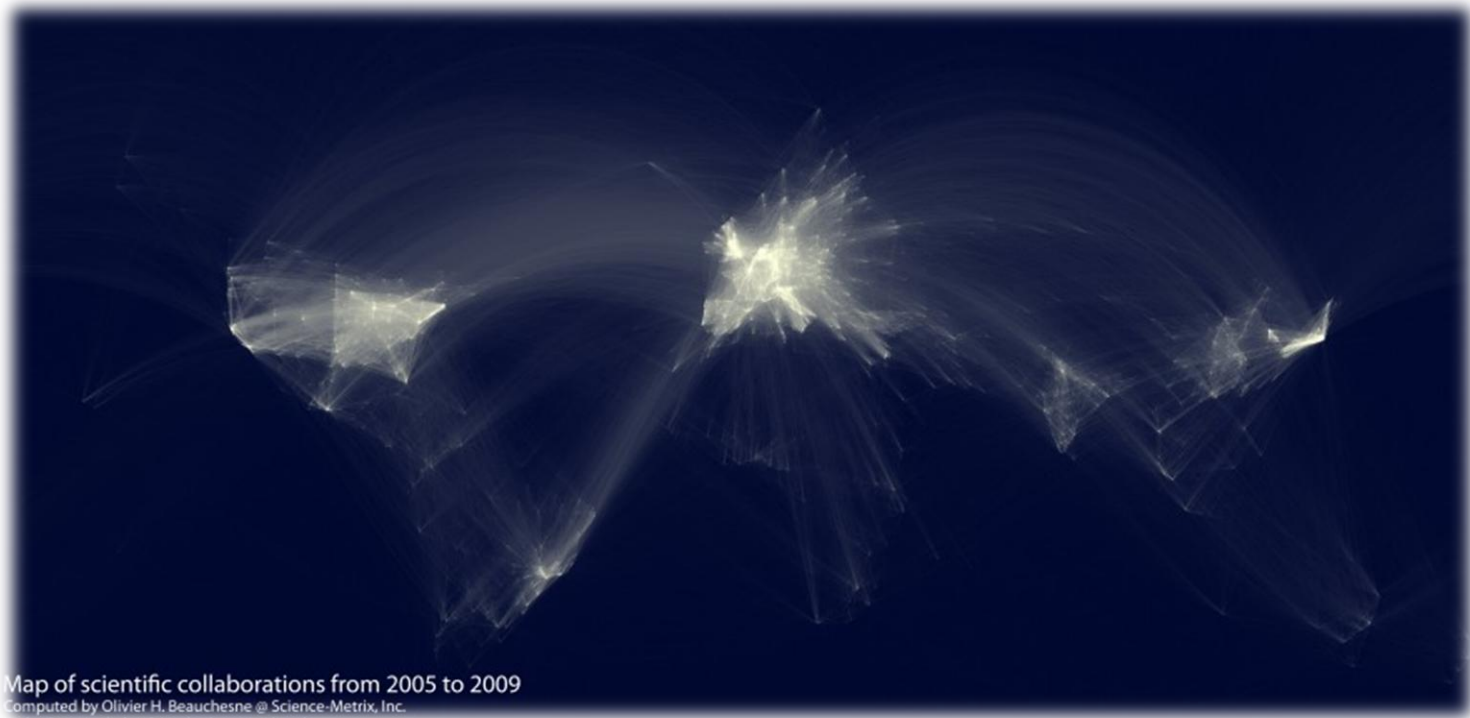
Could a network which is so strongly locally structured be at the same time a small world?

**Yes. You don't need more than a few random links.**



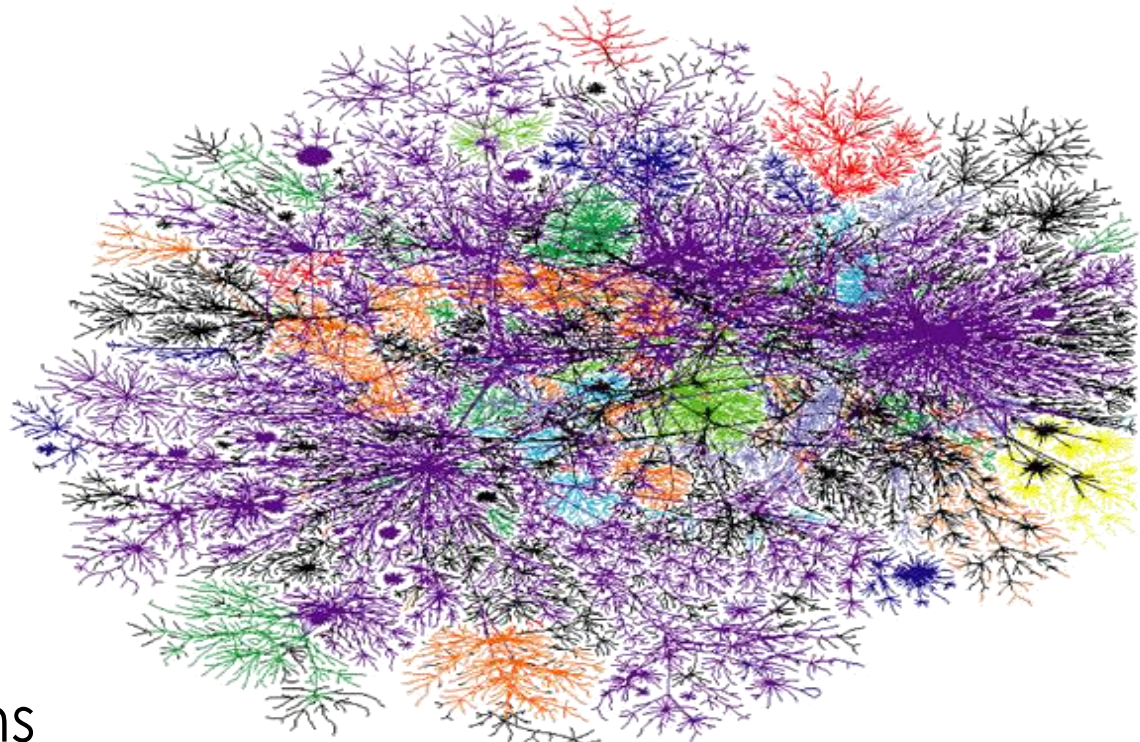


# Map of Scientific Collaborations



# The Internet-based experiment

- 60000 start nodes
- 18 targets
- 384 completed chains
- Average path length between 5 to 7.



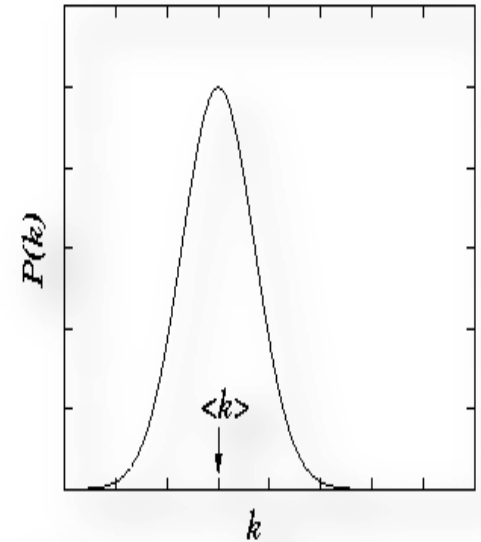
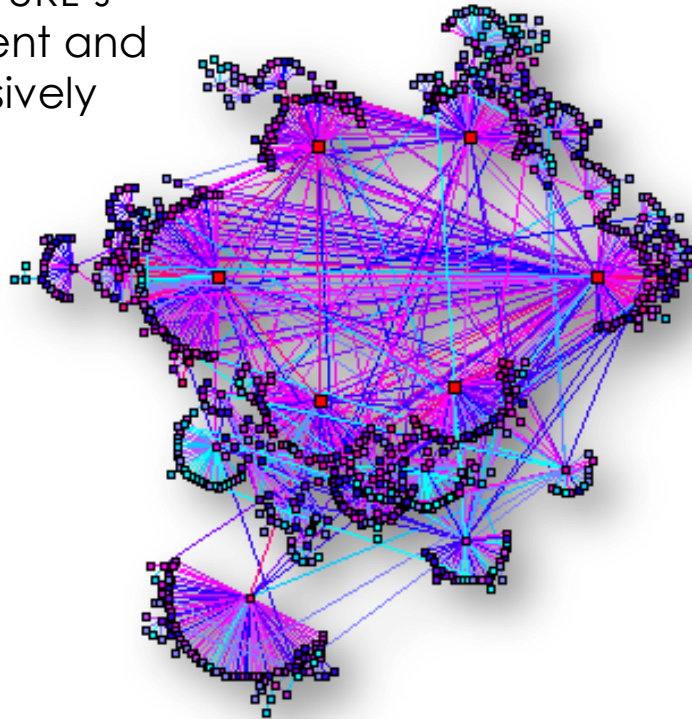
Dodds, Muhamad and Watts, Science **301**,827 (2003)

# How it All Started

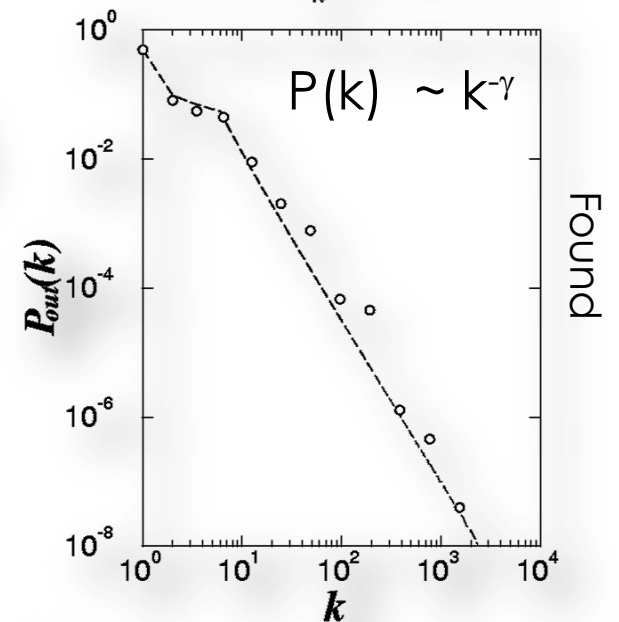
Nodes: WWW documents  
Links: URL links

Over 3 billion documents

ROBOT: collects all URL's  
found in a document and  
follows them recursively



Expected



Found

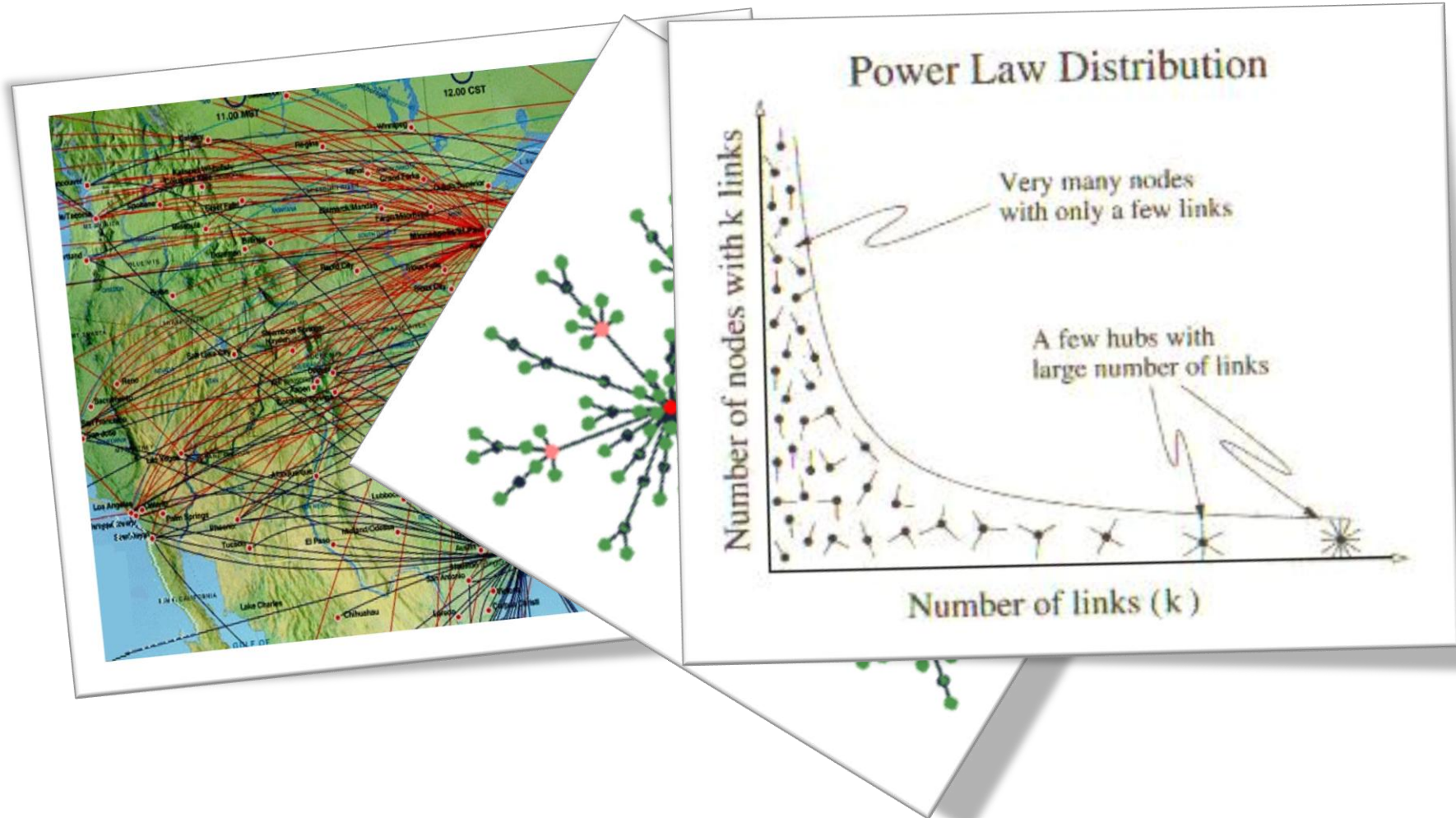
# Scale-free network



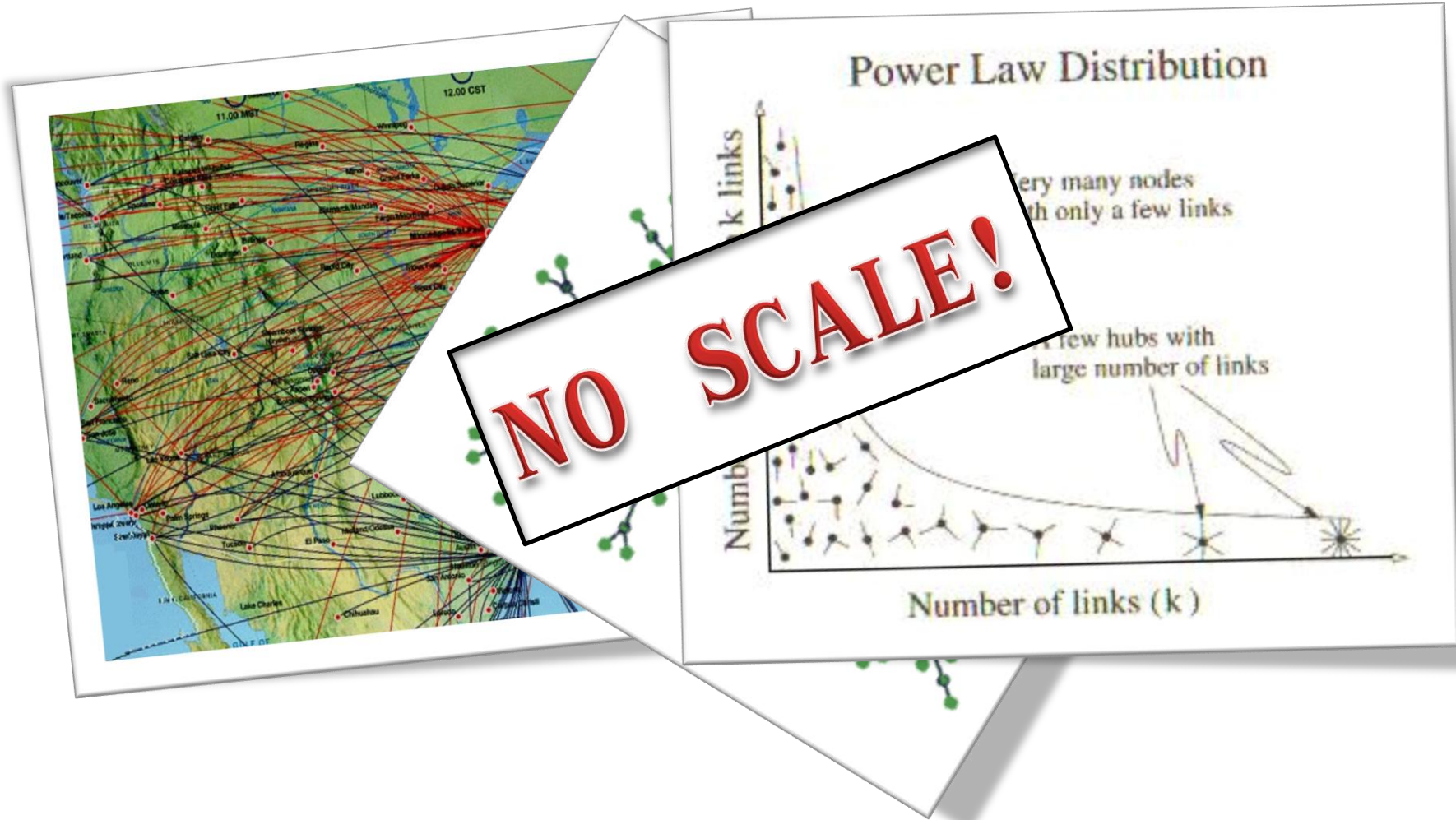
# Scale-free network



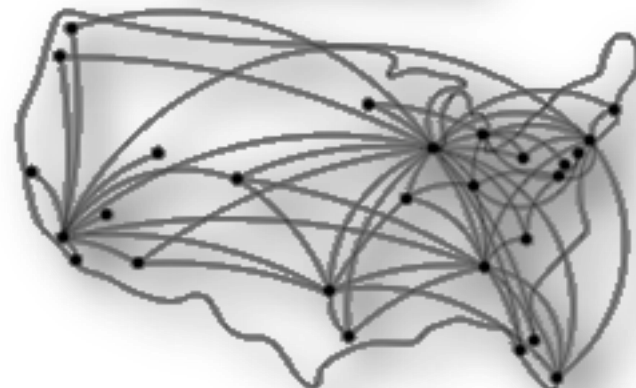
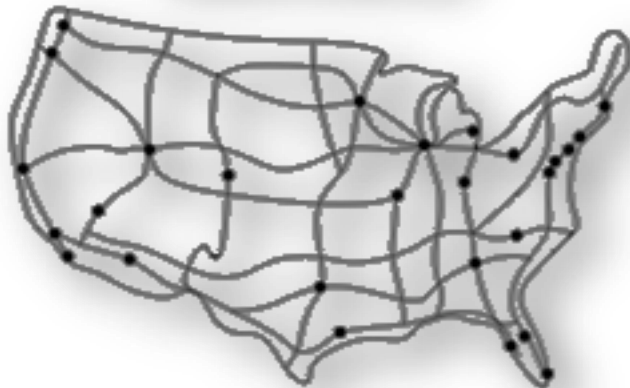
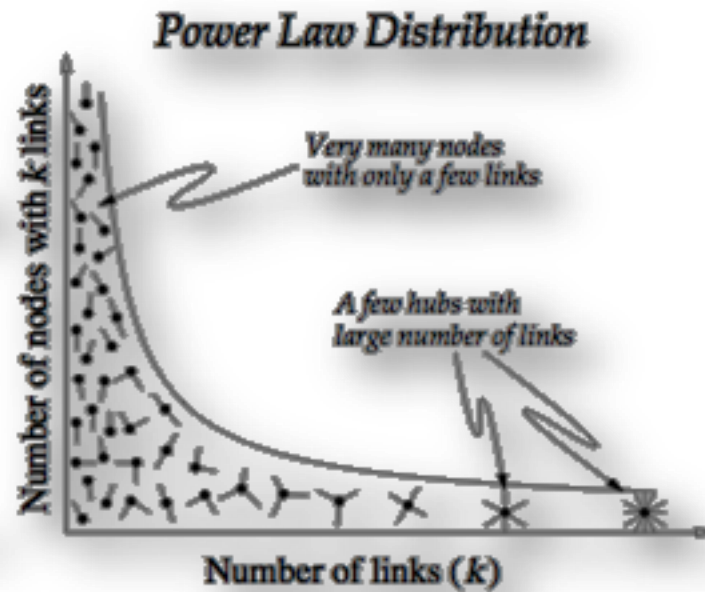
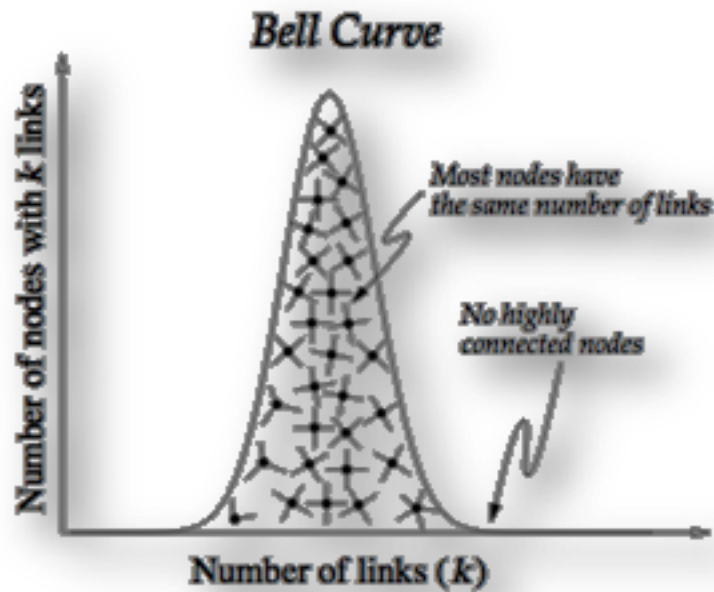
# Scale-free network



# Scale-free network

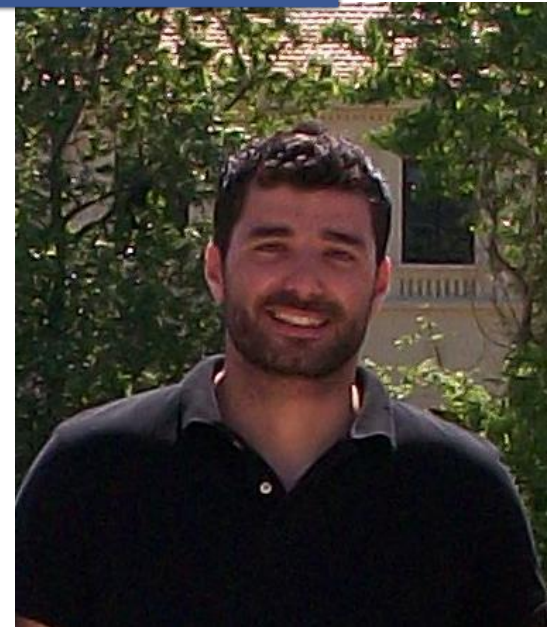


# Scale-free network





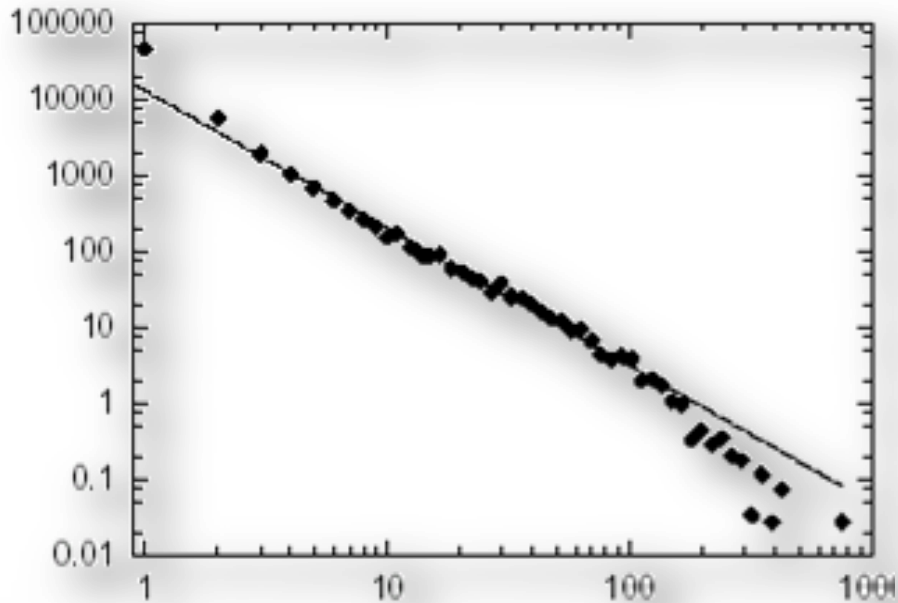
# Scale-free network



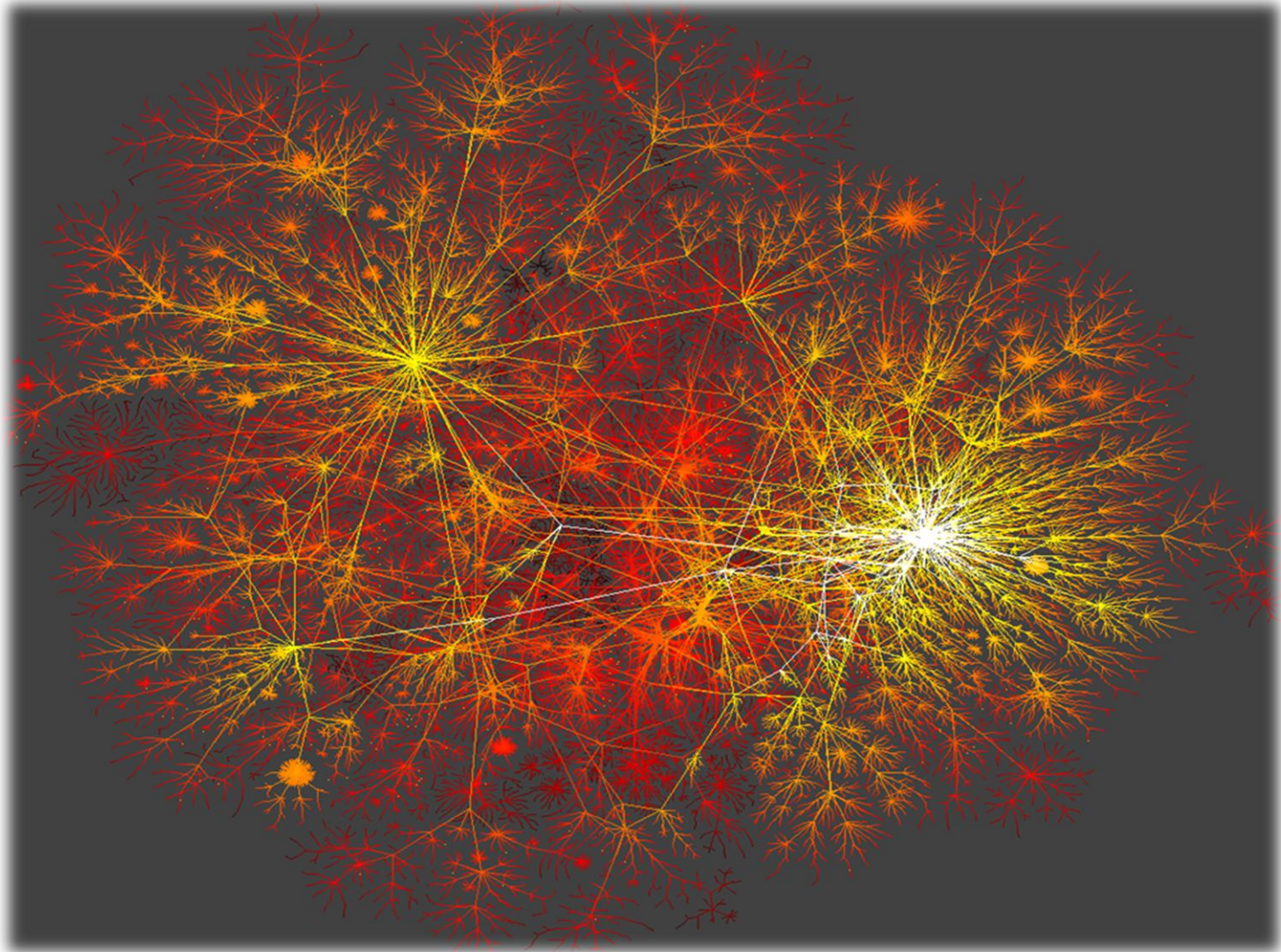
*There are few nodes with a lot of connections..*

*.. and the majority of the nodes have very few connections*

# What it Means to Be a Power-Law

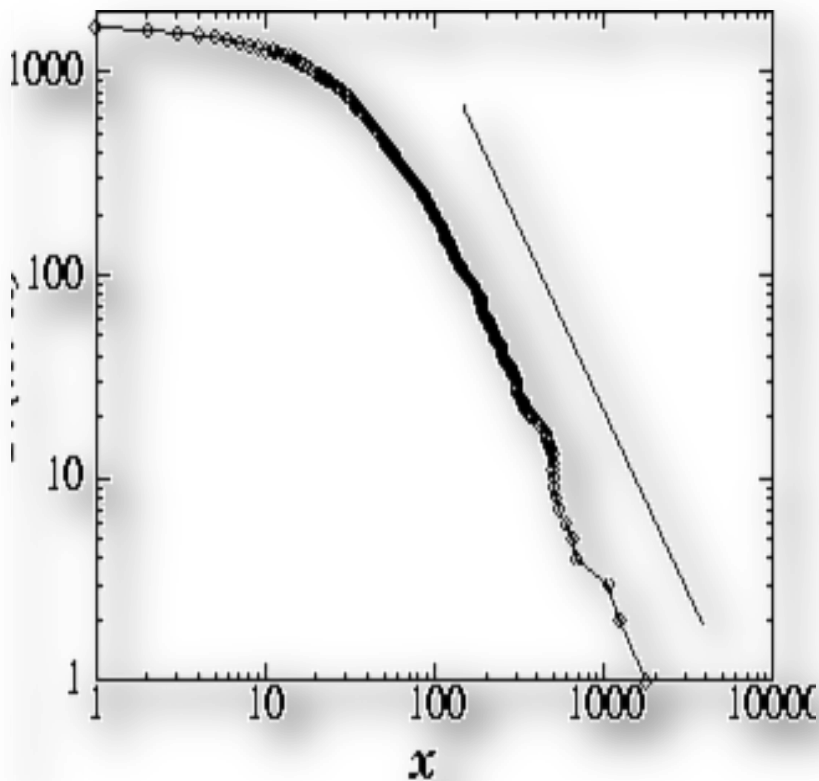


# What it Means to Be a Power-Law



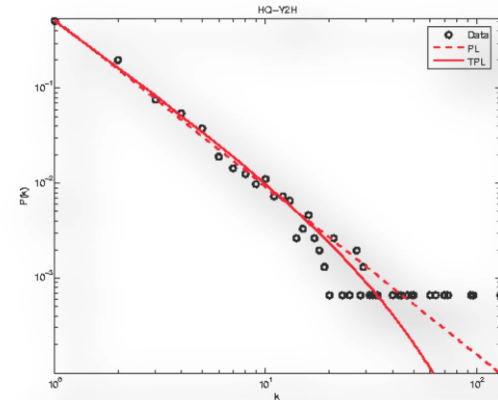
# Power Law Distributions

in Biology



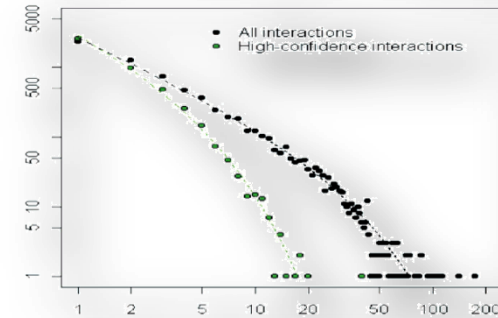
## GENOME

protein-gene interactions



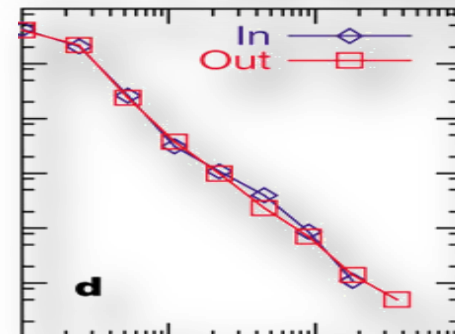
## PROTEOME

protein-protein interactions

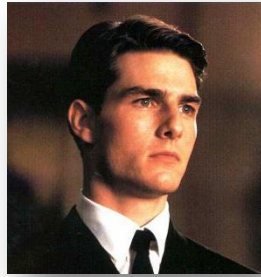


## METABOLISM

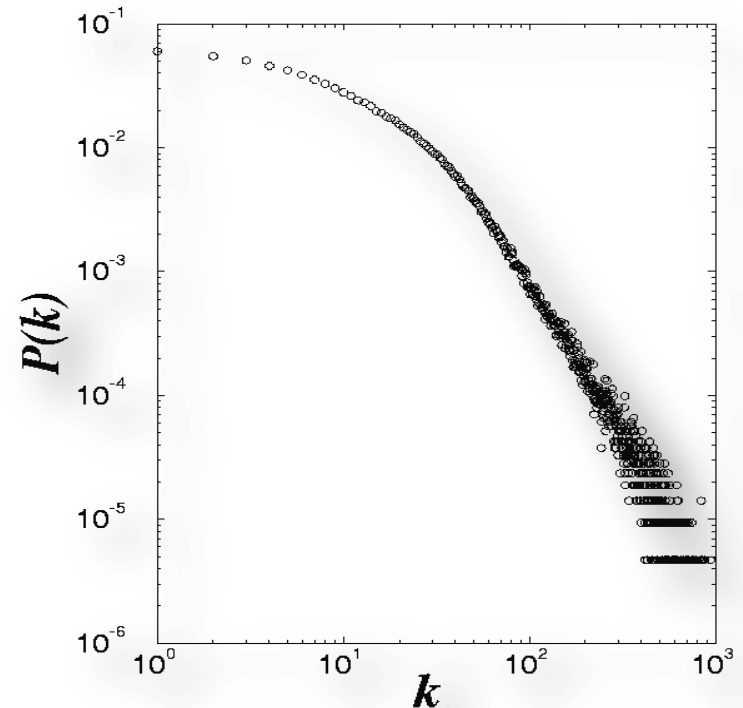
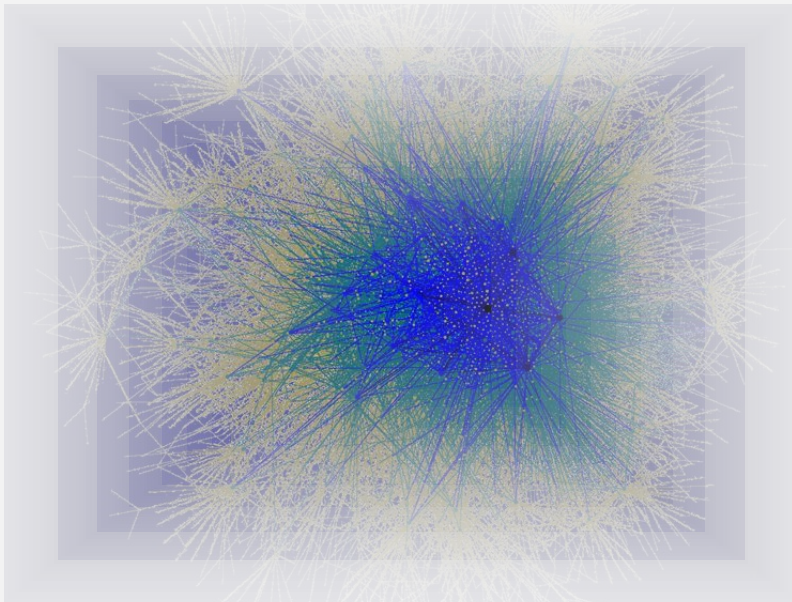
Bio-chemical reactions



# Internet Movie Database - IMDB



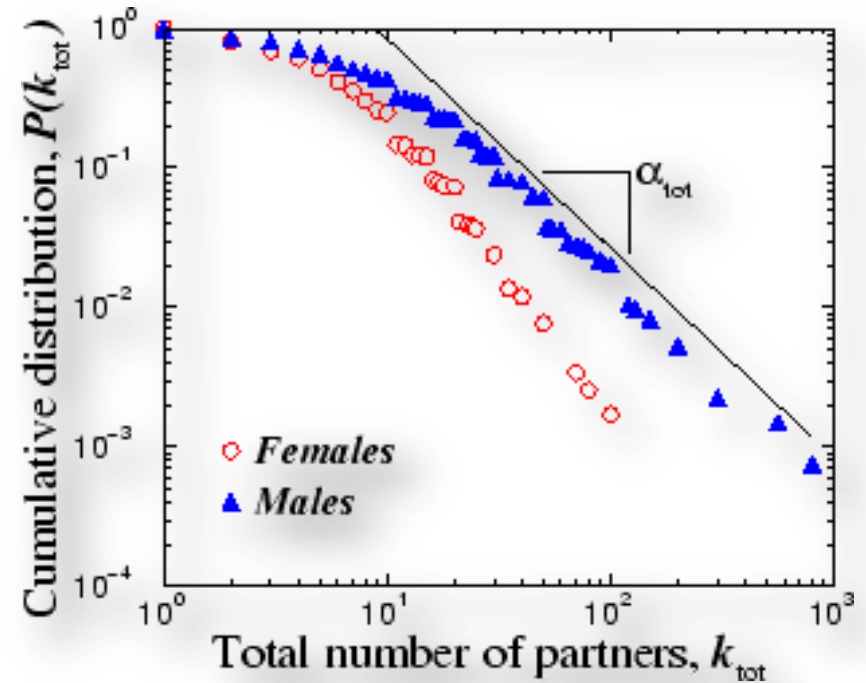
Days of Thunder (1990)  
Far and Away (1992)  
Eyes Wide Shut (1999)



# Sexual Partnership



**Nodes:** people (Females; Males)  
**Links:** sexual relationships



# How to .. Power-Law

## An easy way to have a power-law distribution

1. Generate a random number  $x$  between  $[0,1]$
2. Give to constant  $\gamma$  a standard value
3. Let  $y_{max} = N$  (if you have a Scale-free network of  $N$  nodes)
4. Let  $y_{min} = 1$  (because everyone must be connected to someone!)
5. Use the following equation:

$$y = [(y_{max}^{1-\gamma} - y_{min}^{1-\gamma})x + y_{min}^{1-\gamma}]^{\frac{1}{1-\gamma}}$$

6. Finally  $y$  is the degree of each node of your network
7. Make the distribution of  $p(k)$  in dependence of  $k$
8. Congratulations! You have a Power-Law distribution!

# Power-Law in a computer

Node	degree (k)
1	2
2	1
3	1
4	3
5	2
6	8
7	1
8	2
9	6
10	1

$$y = k$$



$$y_{max} = k_{max}$$



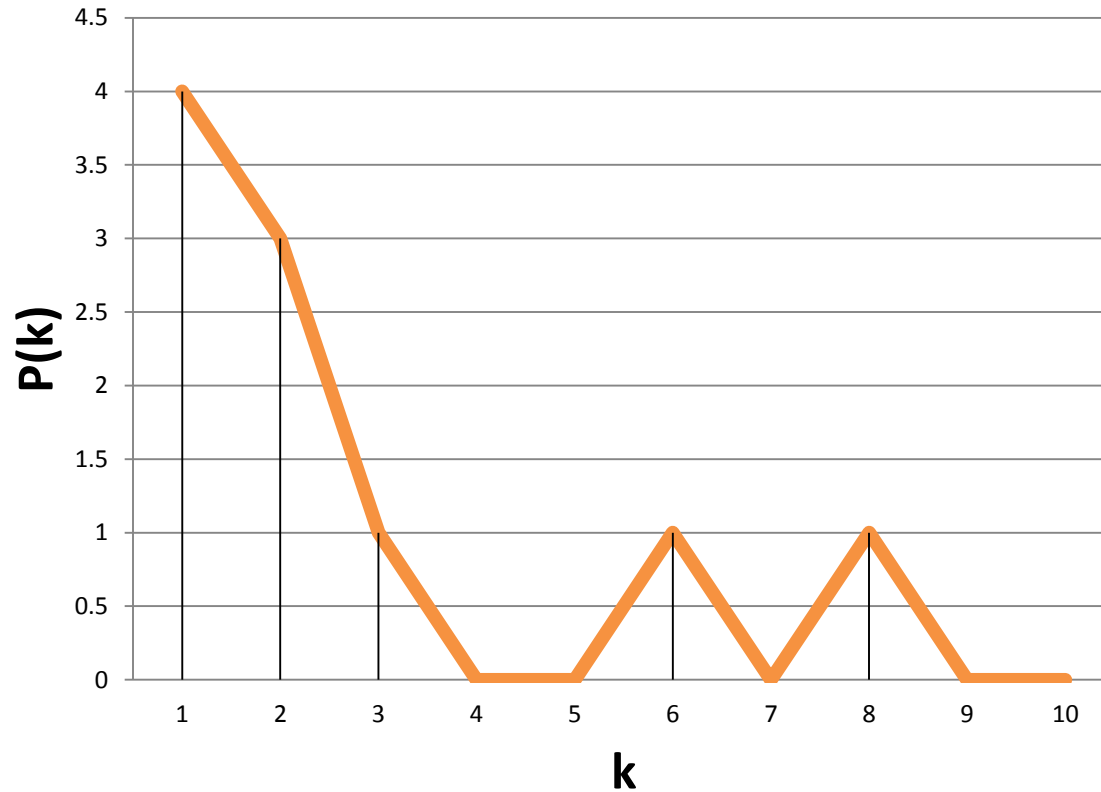
$$y_{min} = k_{min}$$

P(k)	degree (k)
4	1
3	2
1	3
0	4
0	5
1	6
0	7
1	8
0	9
0	10





# Power-Law in a computer



# Exercise 2

## *Scale-Free Network*

Create a network with  $N$  nodes.

Use  $N=10000$  and  $N=100000$  for a Power law distribution which results in a scale-free network.

Use the distribution  $P(k) \sim k^{-\gamma}$ , where  $\gamma$  is constant.

Here use  $k=2, 2.5, 3$ .

Use the values  $k_{\min}=1$  and  $k_{\max}=N$ .

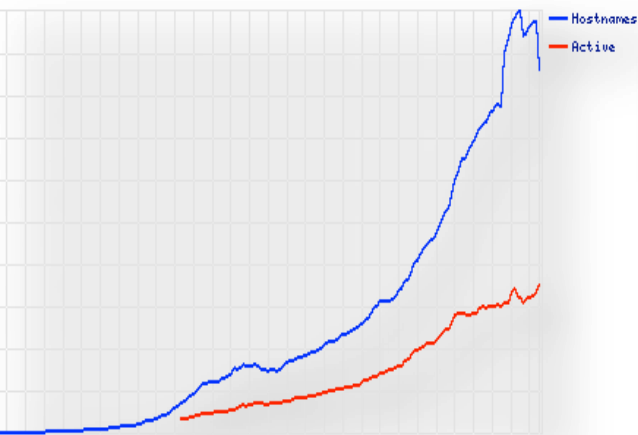
On a graph (with logarithmic axes) create the distributions  $P(k)$  vs  $k$  for the three values of  $\gamma$ .

The data will be the average of 100 runs..

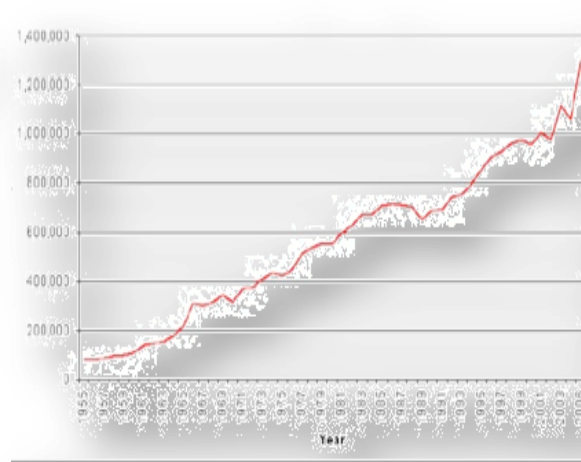
$$y = [(y_{\max}^{1-\gamma} - y_{\min}^{1-\gamma})x + y_{\min}^{1-\gamma}]^{\frac{1}{1-\gamma}}$$

# BA Model – Networks are Not Static

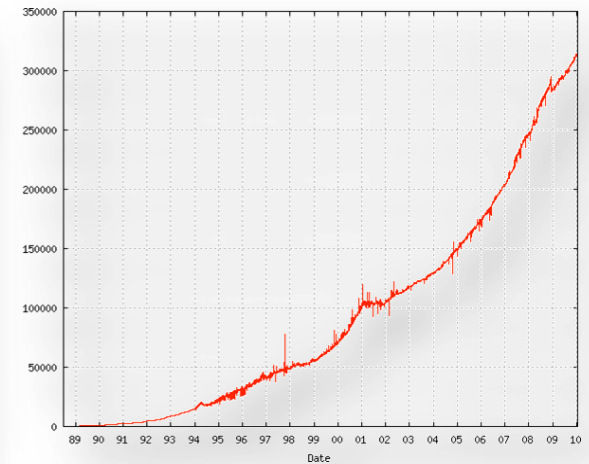
Real networks continuously expand  
by the addition of new nodes



WWW



Citations

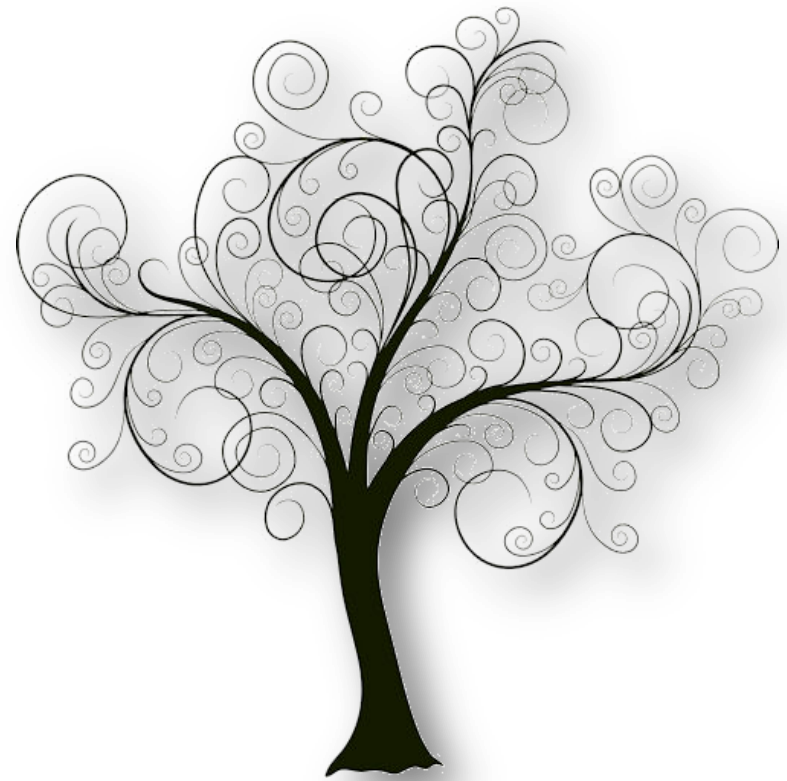
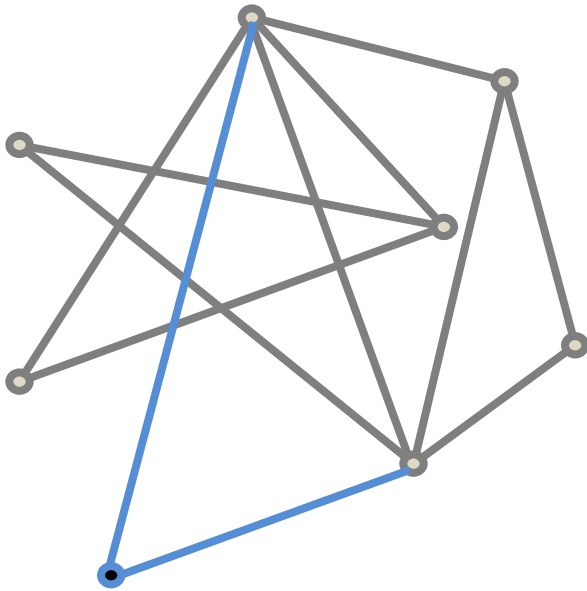


Internet

# BA model – Growth

Networks continuously expand by the addition of new nodes

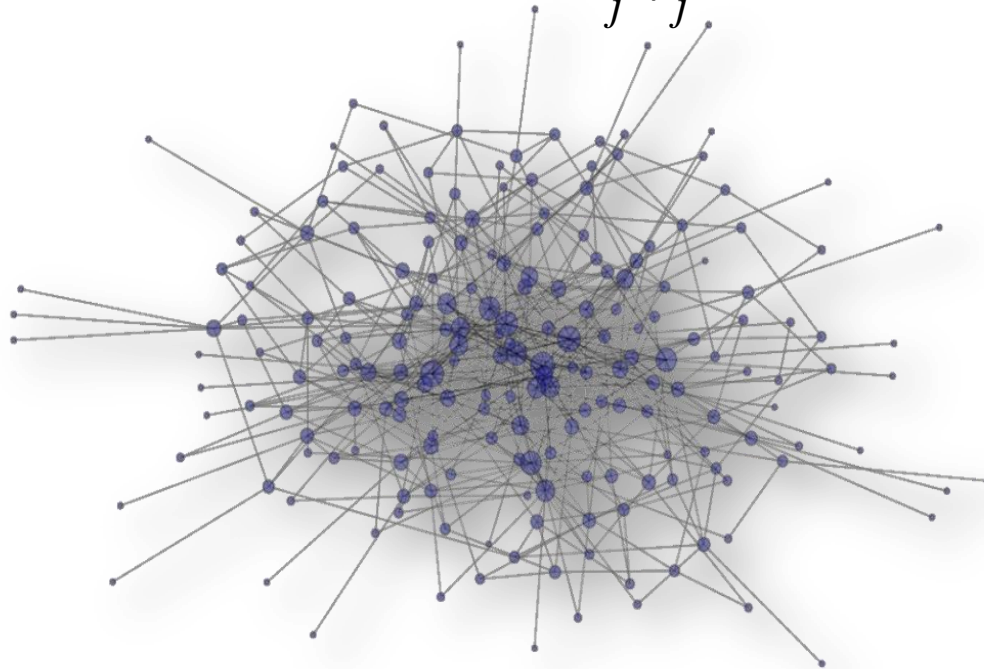
Add a new node with  $m$  links



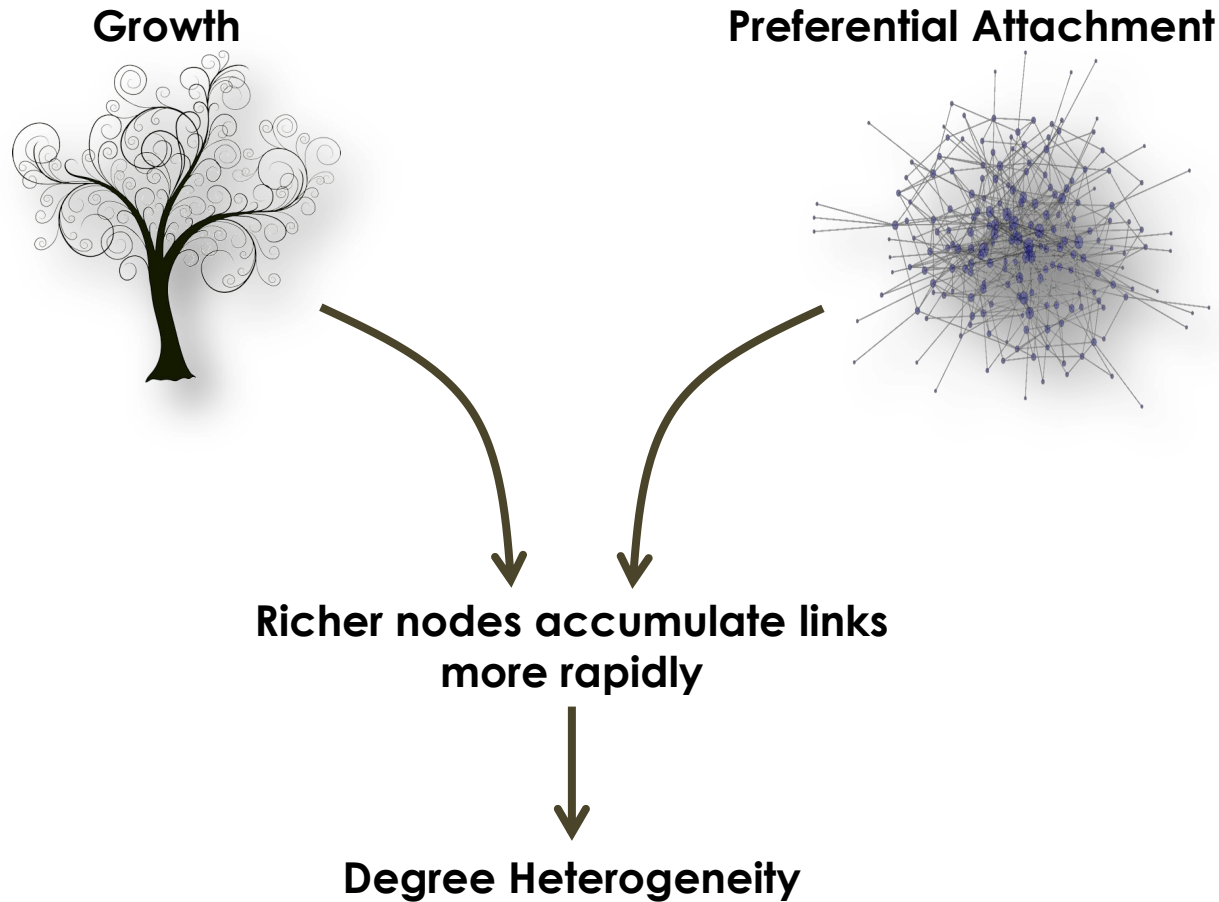
# Preferential attachment

The probability that a node connects to a node with  $k$  links is proportional to  $k$ .

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

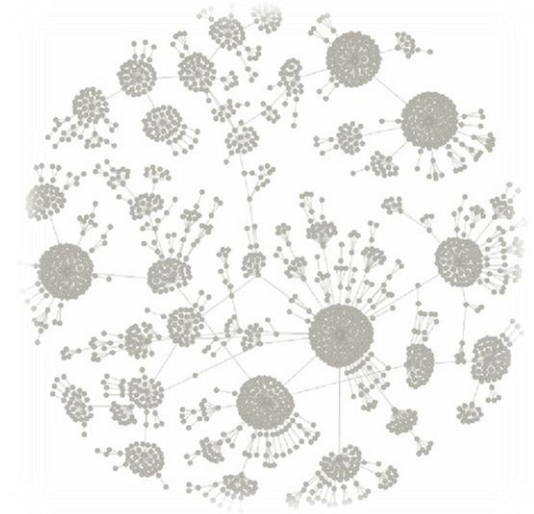
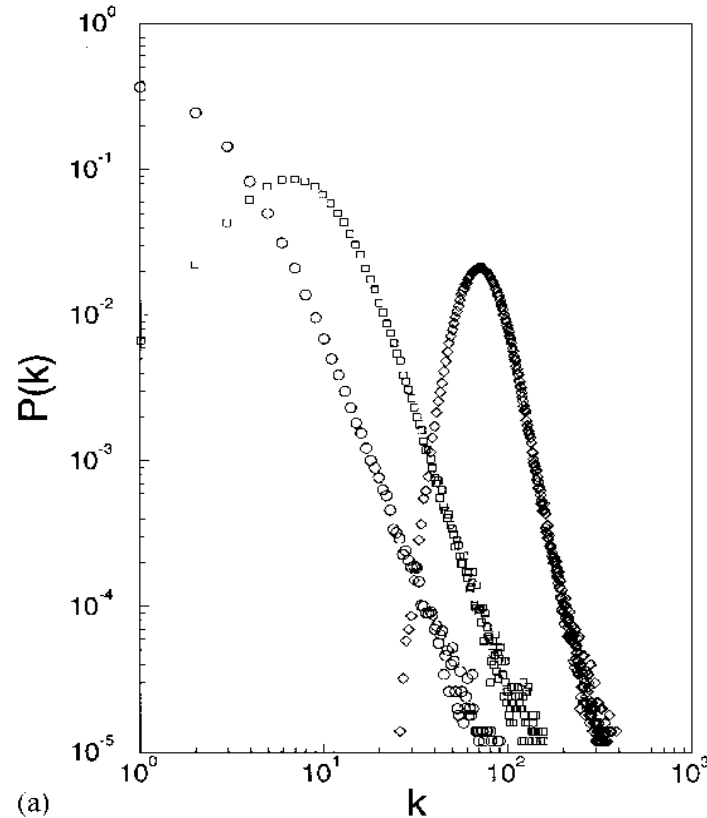


# BA Growth / preferential attachment



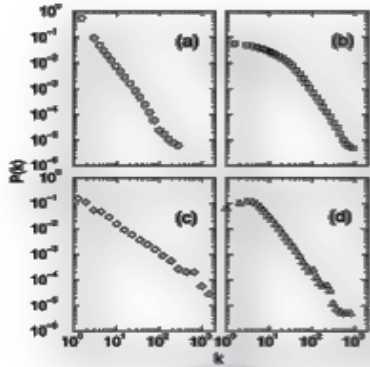
# Power Law

$$P(k) = k^{-\gamma}$$

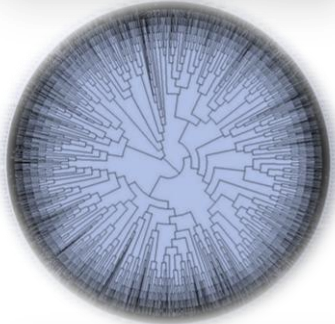


As the power-law describes systems of rather different ages and sizes, it is expected that a correct model should provide a time-independent degree distribution. Indeed, asymptotically the degree distribution of the BA model is independent of time (and of the system size  $N$ ) - the network reaches a stationary scale-free state.

# Universality of Networks



Power law degree distribution



High clustering and community structure



Small world topology

*Despite the diversity in scale, purpose and functionality, the topological characteristics of networks exhibit a high degree of universality*



# Universality of Networks

**Universality** - The observation that there are properties for a large class of systems that are independent of the dynamical details of the system

- *Diverse phenomena explained by the same fundamental principles*
- *Unified set of analytical and empirical tools*
- *Limiting the number of relevant observables*

# Some simulation techniques

```
    nblocks = (gidsetsize + NGROUPS_PER_BLOCK - 1) / NGROUPS_PER_BLOCK;
    /* Make sure we always allocate at least one indirect block pointer */
    nblocks = nblocks ? : 1;
    group_info = kmalloc(sizeof(*group_info) + nblocks*sizeof(gid_t *), GFP_USER);
    if (!group_info)
        return NULL;
    group_info->ngroups = gidsetsize;
    group_info->nblocks = nblocks;
    atomic_set(&group_info->usage, 1);

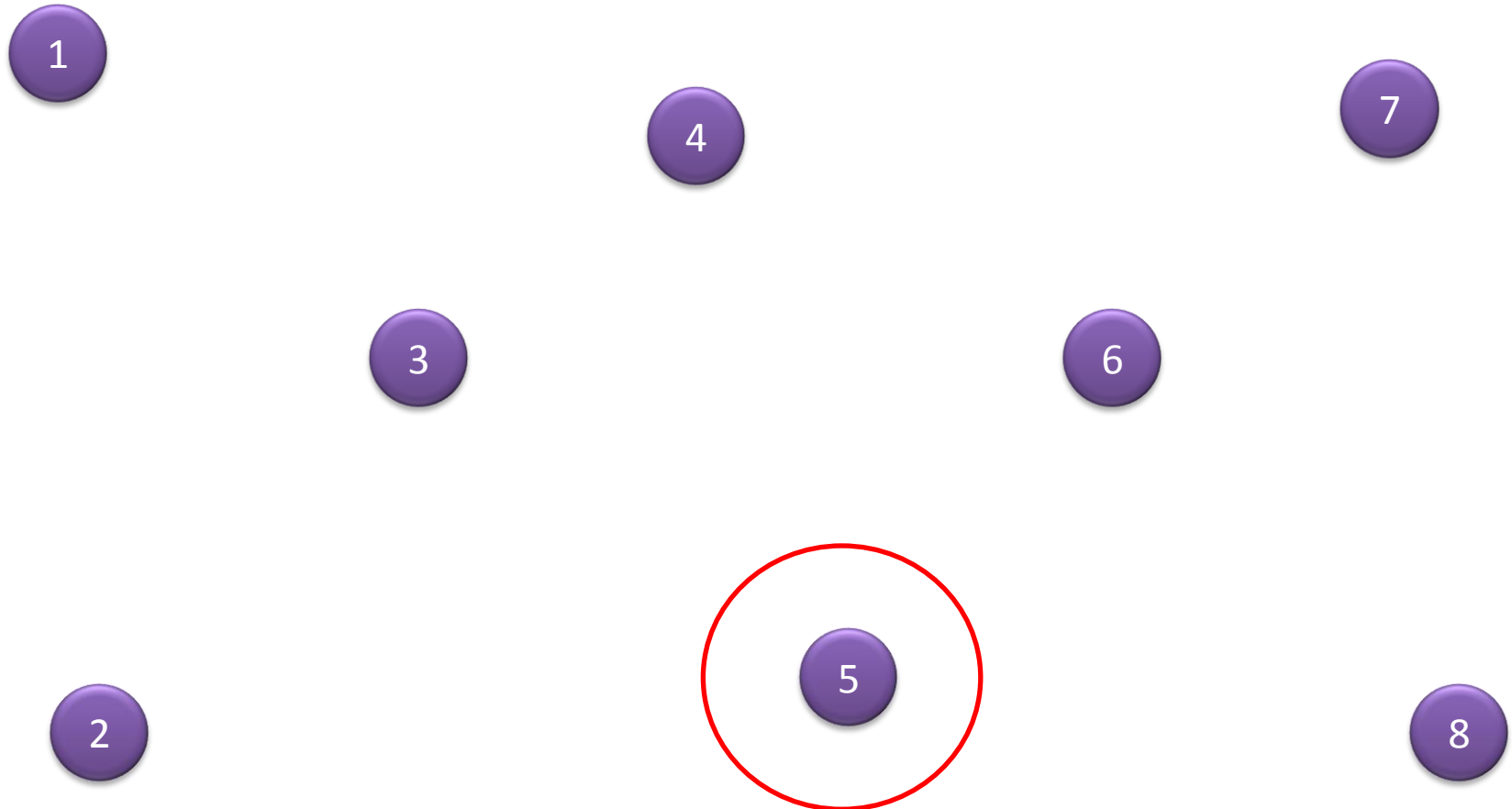
    if (gidsetsize <= NGROUPS_SMALL)
        group_info->blocks[0] = group_info->small_block;
    else {
        for (i = 0; i < nblocks; i++) {
            gid_t *b;
            b = (void *)__get_free_page(GFP_USER);
            if (!b)
                goto out_undo_partial_alloc;
            group_info->blocks[i] = b;
        }
    }
    out_undo_partial_alloc:
    if (group_info->nblocks > 0)
        kfree(group_info->blocks);
    kfree(group_info);
    return NULL;
}
```

```
    group_info->blocks[i] = p;
    goto out_undo_partial_alloc;
}
if (ip)
    p = (void *)__get_free_page(GFP_USER);
return p;
}
```

# Some simulation techniques

Step 1:

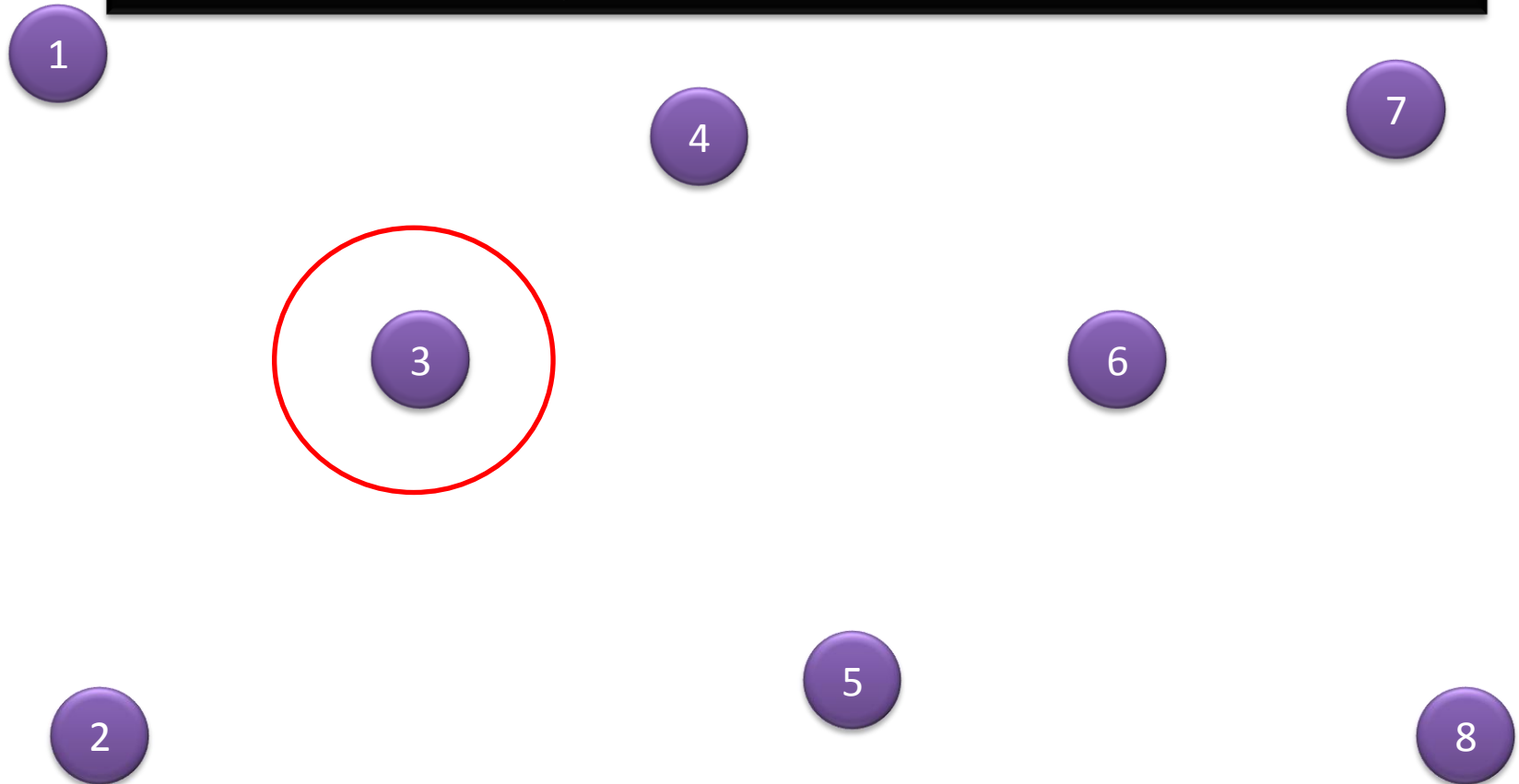
We generate a random number  $x$  between  $[1,8]$ , representing the node of the network which is to be connected.



# Some simulation techniques

Step 2:

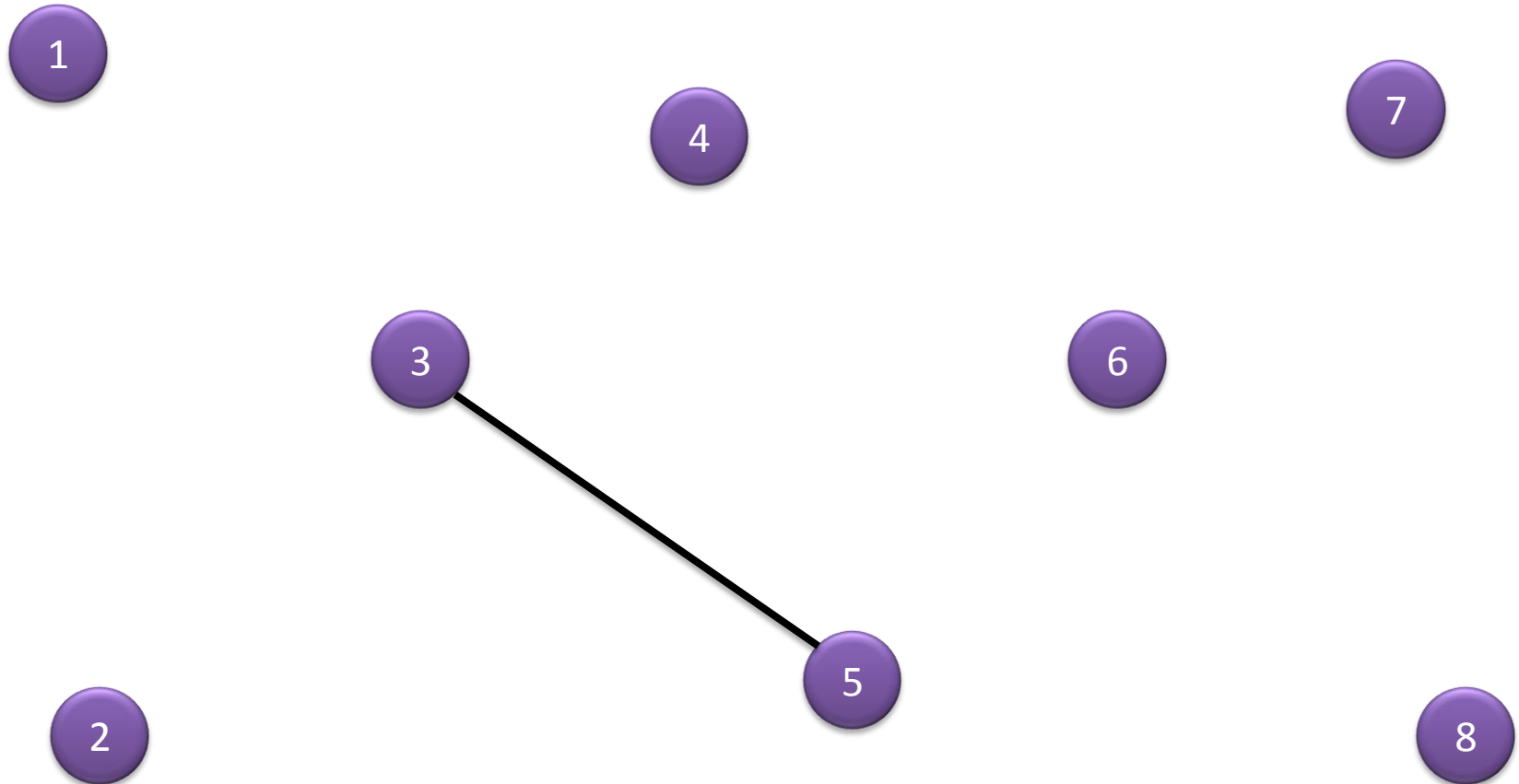
We generate then a random number  $x_2$  between  $[1,8]$ , representing the node of the network which is to be connected with the previous one.



# Some simulation techniques

Step 3:

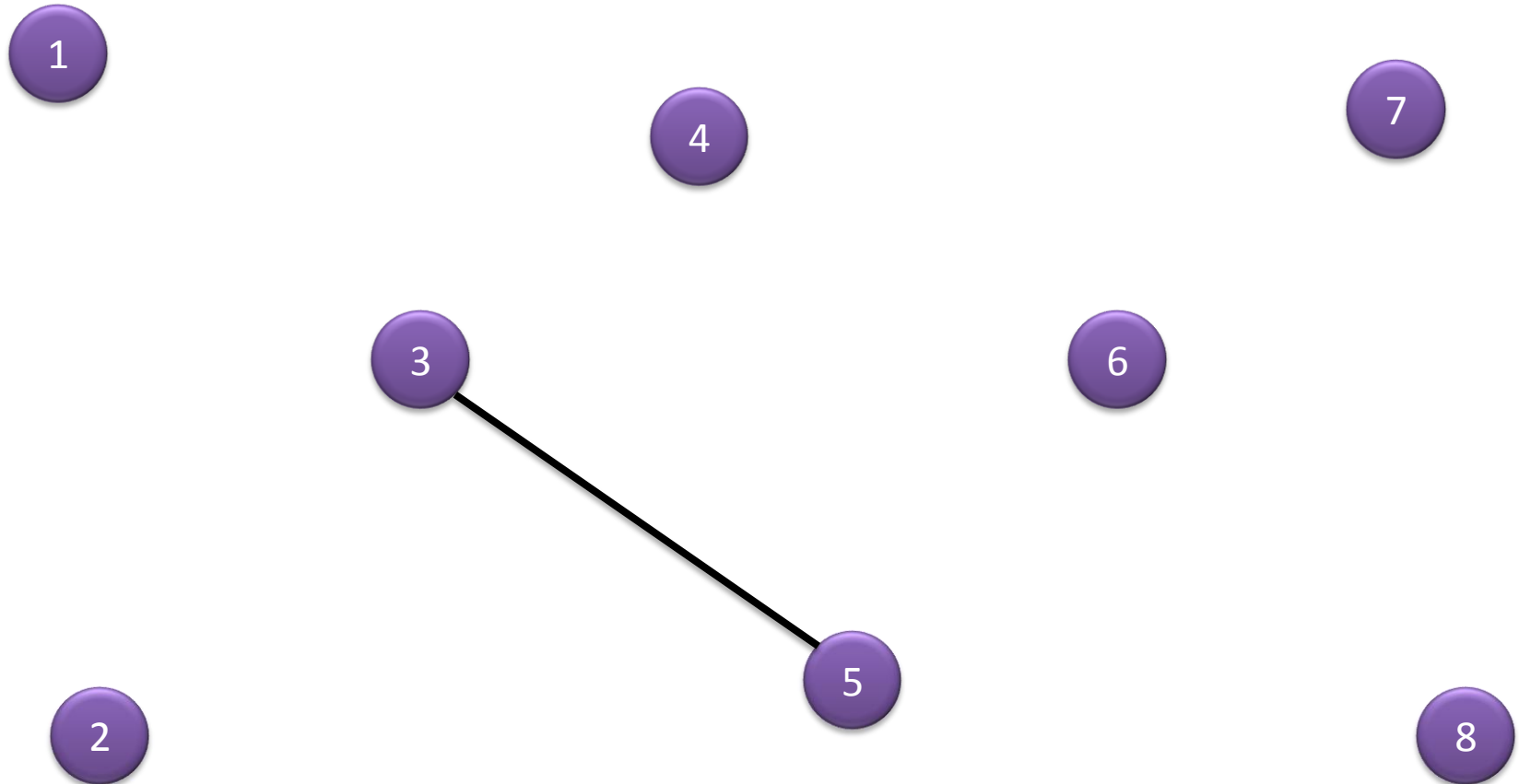
We consider the two nodes as connected (we put the link).



# Some simulation techniques

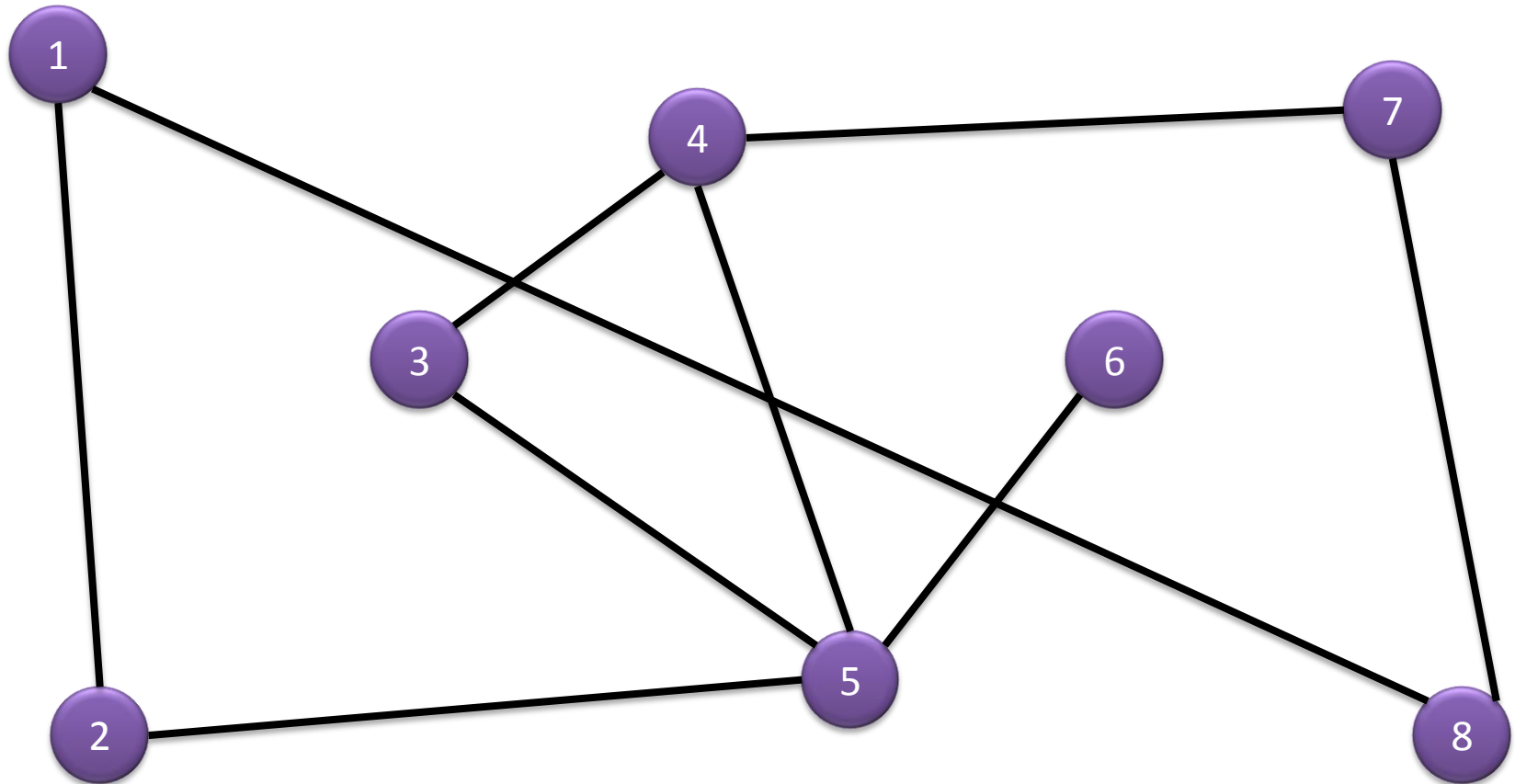
Step 4:

We repeat the previous 3 steps as much as we want..



# Some simulation techniques

Simulating a network



# Some simulation techniques

Simulating a network all we need is a two-dimensional array

nodes	1	2	3	4	5	6	7	8
1	0	1	0	0	0	0	0	1
2	1	0	0	0	1	0	0	0
3	0	0	0	1	1	0	0	0
4	0	0	1	0	1	0	1	0
5	0	1	1	1	0	1	0	0
6	0	0	0	0	1	0	0	0
7	0	0	0	1	0	0	0	1
8	1	0	0	0	0	0	1	0





# for more information...

- *visit:*

- <http://icoscis.physics.auth.gr>

- <http://kelifos.physics.auth.gr/COURSES/courses.html>



- *or e-mail:*

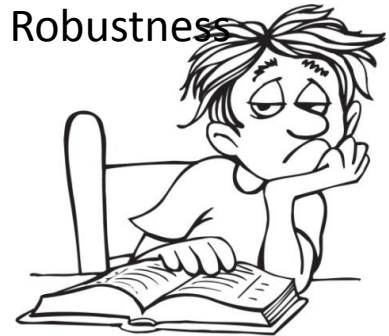
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- *Or search for .. :*

1. D.P. Landau and K. Binder, “A Guide to Monte Carlo Simulations in Statistical Physics”, Cambridge University Press, 2000
2. Yaneer Bar-Yan, “Dynamics of complex systems”, Addison – Wesley, 1997
3. S. Solomon and E. Shir, “Complexity; a science at 30”, europhysics news, March/April 2003
4. G.H.Weiss, “Aspects and Applications of the Random Walk”, North Holland 1994
5. Dietrich Stauffer : “Introduction to percolation theory”, Taylor & Francis, 1985
6. Reuven Cohen & Shlomo Havlin, “Complex Networks: Structure, Robustness and Function”, Cambridge University Press, 2010



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<http://db.tt/9PdUuOrq>



***Thank you for your attention!!***

